SOLID STATE TESLA COIL

by

Dr. Gary L. Johnson
Manhattan, Kansas

Some years ago I developed an interest in Tesla coils. I was teaching a senior elective course at Kansas State University where we talked about power MOSFETs and topics related to high voltages and currents. I decided to use a Tesla coil as a class project. We would talk about design aspects, then design, build, and test a coil. The best description of the results of that plan was fiasco, or maybe disaster. We had some sparks, but none where they belonged. That was one of the most humiliating experiences of my career.

I learned several things from that experience. One is that the Tesla coil is more complex than I had thought. Another was that there seemed to be a mismatch between theory and experiment. At that time, at least, people would go through pages of high powered mathematics and quit without giving an example of how to use all the formulas. Experimentalists would sometimes make fun of the theorists, and give rules-of-thumb on how to make long sparks. It was like I was hearing a debate on whether the best cooks use recipes or not. My mother never used a recipe and I always enjoyed her cooking. However, my own talents are such that if I am to cook anything fit to eat, I need a recipe.

This book is written for people like me, challenged when faced with doing something without a recipe or complete set of instructions. I will throw in things learned from other Tesla coil builders, but will quickly admit that when it comes to making long sparks, there are many who are far better than I.

I started asking questions about Tesla coils that any electrical engineer would ask. These include:

1. What is the input impedance?
2. What are the fractions of input power that are dissipated in the spark itself, in electromagnetic radiation, the coil wire, the coil form, the toroid, the spark gap, and other
circuit components?

3. Are there circuit models that allow these questions to be answered on the computer before building and testing devices in open air?

4. What are the differences between Tesla coils driven by or through spark gaps, vacuum tubes, or solid state devices?

5. What are the important factors in producing long sparks (energy per bang, power input at spark inception, rate of change of power, the coil, the toroid, etc.)?

One would expect the answers to these questions to come from a mix of theory and experiment. One would develop a theory or model and then go to the laboratory to measure parameters and check performance. The theory would then be adjusted to reflect experimental observations.

We now review a little Tesla coil history and look at the ‘simplest’ model, the lumped circuit element model.

1 History

Nikola Tesla (1856 - 1943) was one of the most important inventors in human history. He had 112 U.S. patents and a similar number of patents outside the United States, including 30 in Germany, 14 in Australia, 13 in France, and 11 in Italy. He held patents in 23 countries, including Cuba, India, Japan, Mexico, Rhodesia, and Transvaal. He invented the induction motor and our present system of three-phase power in 1888 [20]. He invented the Tesla coil, a resonant air-core transformer, in 1891. Then in 1893, he invented a system of wireless transmission of intelligence. Although Marconi is commonly credited with the invention of radio, the U.S. Supreme Court decided in 1943 that the Tesla Oscillator patented in 1900 had priority over Marconi’s patent which had been issued in 1904 [15]. Therefore Tesla did the fundamental work in both power and communications, the major areas of electrical engineering. These inventions have truly changed the course of human history.

After Tesla had invented three-phase power systems and wireless radio, he turned his attention to further development of the Tesla coil. He built a large laboratory in Colorado Springs in 1899 for this purpose. The Tesla secondary was about 51 feet in diameter. It was in a wooden building in which no ferrous metals were used in construction [15]. There was a massive 80-foot wooden tower, topped by a 200-foot mast on which perched a large copper ball which he used as a transmitting antenna. The coil worked well. There are claims of bolts of artificial lightning over a hundred feet long, although Richard Hull asserts that from Tesla’s notes, he never claimed a distance greater than 43 feet. From photographic evidence, the maximum may have been closer to 22 feet [12].
Tesla then abandoned the Colorado Springs Laboratory early in 1900, having learned what he needed from that facility, and also having become somewhat unpopular as a result of frequently knocking the local sub-station off line.

Since that time, it appears that no one has built a Tesla coil of both the size and performance of the Colorado Springs coil. Apparently the only coil of that size was built by Robert Golka at Wendover Air Force Base in Utah [8] and later moved to a facility near Leadville, Colorado [9, 19]. The original purpose of this coil was to produce artificial lightning for testing the effects of lightning striking aircraft in flight. Golka determined that the average voltage produced in Utah was about 10 MV, with the highest voltage observed being 25 MV. Operation was spectacular, even if not quite at the level of the Colorado Springs coil.

When Golka’s coil was moved to Leadville, however, it performed very poorly. Golka and his associates were basically unable to properly tune the coil. There has been considerable speculation over the reasons for the difference in performance, but one problem seems to be that we did not have adequate theoretical models for the design and operation of Tesla coils. What appeared to be minor differences in location and construction caused a major decrease in performance. The number of variables was simply too large to allow for a purely experimental optimization of performance before the coil was dismantled and moved early in 1990.

Some work on theoretical models has been performed by high energy physicists [6, 10, 1, 17, 18]. They are interested in high voltage capacitor discharges for research in plasma physics and in the production of pulsed particle or radiation beams. The most common way of producing such high voltage discharges is the Marx circuit, in which capacitors are charged in parallel to a lower voltage and then discharged in series through a number of airgaps. The Marx circuit requires the capacitor bank to be divided into sub-banks well-insulated from each other and from ground. A Tesla coil offers an alternative method of charging the high voltage capacitors. Discharges are reported in the range of 100 kA at 1 MV, with one report of 2.5 MV [10]. These models are all lumped parameter models.

There are a number of experimenters who build Tesla coils as a hobby. The Tesla Coil Builders Association has several hundred members and a quarterly newsletter published by Harry Goldman [7]. Harry has announced plans to stop publishing the newsletter at the end of 2001. The Tesla Coil Builders of Richmond has been a very active local group [11], although their leader Richard Hull has recently become interested in other activities. A number of manuals are available on how to build coils [16, 4, 5]. The one by Lee [16] is especially well illustrated with pictures of capacitors and other components that might be needed for a moderate sized Tesla coil. There is an Internet listserv (www.pupman.com) that has about 700 subscribers, which has been very helpful to me.

The brothers James and Kenneth Corum have done considerable work on distributed models of Tesla coils in the past few years [2, 3]. They argue that lumped parameter models are not adequate for all situations. Sometimes a distributed circuit analysis must be made. In this case, the Tesla coil secondary and another component called the extra coil are considered
as sections of transmission lines. This explains some of the effects in an elegant manner. They have written a sophisticated computer program, TCTUTOR, to analyze Tesla coils. They have also performed considerable historical research into Tesla’s notes made on his facility in Colorado Springs [21].

The Tesla coil community is divided over the issue of lumped versus distributed models. A majority favors the lumped model approach. Some are outspoken in their belief that distributed models are useless at best and just plain wrong on important issues. I confess to being somewhere in the middle on this controversy. James Corum and I both have our Ph.D.s in electromagnetic theory, so I can mostly understand what he says, and I therefore have a natural orientation to the distributed approach. In my eyes, I am like a Baptist pastor of a 50 person congregation and James is like Billy Graham. That is, I hold him in awe. I have heard the Corums speak several times, and have gotten caught up in their knowledge and excitement.

On the other hand, I cannot honestly say that TCTUTOR has been helpful to me in building and understanding Tesla coils. I can see significant problems with distributed models, which will be discussed later. And James, like many bright people, has a tendency to talk down to us slow ones. This puts some people off, of course.

In this book we will look at both lumped and distributed models. We will point out difficulties with both. We will look at some data, and ask which approach does best in describing reality.

2 Classical Tesla Coil

A classical Tesla coil contains two stages of voltage increase. The first is a conventional iron core transformer that steps up the available line voltage to a voltage in the range of 12 to 50 kV, 60 Hz. The second is a resonant air core transformer (the Tesla coil itself) which steps up the voltage to the range of 200 kV to 1 MV. The high voltage output is at a frequency much higher than 60 Hz, perhaps 500 kHz for the small units and 80 kHz (or less) for the very large units.

The lumped circuit model for the classical Tesla coil is shown in Fig. 1. The primary capacitor $C_1$ is a low loss ac capacitor, rated at perhaps 20 kV, and often made from mica or polyethylene. The primary coil $L_1$ is usually made of 4 to 15 turns for the small coils and 1 to 5 turns for the large coils. The secondary coil $L_2$ consists of perhaps 50 to 400 turns for the large coils and as many as 400 to 1000 turns for the small coils. The secondary capacitance $C_2$ is not a discrete commercial capacitor but rather is the distributed capacitance between the windings of $L_2$ and the voltage grading structure at the top of the coil (a toroid or sphere) and ground. This capacitance changes with the volume charge density around the secondary, increasing somewhat when the sparks start. It also changes with the surroundings of the coil, increasing as the coil is moved closer to a metal wall. This may have been one of the reasons
that Golka’s coil worked better in Utah than in Colorado, because the metal walls were closer to the coil in Colorado.

Figure 1: The Classical Tesla Coil

The symbol $G$ represents a spark gap, a device which will arc over at a sufficiently high voltage. The simplest version is just two metal spheres in air, separated by a small air gap. It acts as a voltage controlled switch in this circuit. The open circuit impedance of the gap is very high. The impedance during conduction depends on the geometry of the gap and the type of gas (usually air), and is a nonlinear function of the current density. This impedance is not negligible. A considerable fraction of the total input power goes into the production of light, heat, and chemical products at the spark gap. In any complete analysis for efficiency, an equivalent gap resistance $R_{\text{gap}}$ could be defined such that $i^2 R_{\text{gap}}$ would represent the power loss in the gap. This would have rather limited usefulness because of the mathematical difficulty of describing the arc.

The arc in the spark gap is similar to that of an electric arc welder in visual intensity. That is, one should not stare at the arc because of possible damage to the eyes. At most displays of classical Tesla coils, the spark gap makes more noise and produces more light than the electrical display at the top of the coil.

When the gap is not conducting, the capacitor $C_1$ is being charged in the circuit shown in Fig. 2, where just the central part of Fig. 1 is shown. The inductive reactance is much smaller than the capacitive reactance at 60 Hz, so $L_1$ appears as a short at 60 Hz and the capacitor is being charged by the iron core transformer secondary.

A common type of iron core transformer used for small Tesla coils is the neon sign transformer (NST). Secondary ratings are typically 9, 12, or 15 kV and 30 or 60 mA. An NST has a large number of turns on the secondary and a very high inductance. This inductance will limit the current into a short circuit at about the rated value. An operating neon sign has a low impedance, so current limiting is important to long transformer life. However, in Tesla coil use, the NST inductance will resonate with $C_1$. The NST may supply two or three time the NST rated current in this application. Overloading the NST produces longer sparks, but
may also cause premature failure.

When the voltage across the capacitor and gap reaches a given value, the gap arcs over, resulting in the circuit in Fig. 3. We are not interested in efficiency in this introduction so we will model the arc as a short circuit. The shorted gap splits the circuit into two halves, with the iron core transformer operating at 60 Hz and the circuit to the right of the gap operating at a frequency (or frequencies) determined by $C_1$, $L_1$, $L_2$, and $C_2$. It should be noted that the output voltage of the iron core transformer drops to (approximately) zero while the input voltage remains the same, as long as the arc exists. The current through the transformer is limited by the transformer equivalent series impedance shown as $R_s + jX_s$ in Fig. 3. As mentioned, this operating mode is not a problem for the NST. However, the large Tesla coils use conventional transformers with per unit impedances in the range of 0.05 to 0.1. A transformer with a per unit impedance of 0.1 will experience a current of ten times rated while the output is shorted. Most transformers do not survive very long under such conditions. Golka was not alone in burning out some of his transformers. The solution is to include additional reactance in the input circuit.

The equivalent lumped circuit model of the Tesla coil while the gap is shorted is shown in Fig. 4. $R_1$ and $R_2$ are the effective resistances of the air cored transformer primary and secondary, respectively. The mutual inductance between the primary and secondary is shown...
by the symbol $M$. The coefficient of coupling is well under unity for an air cored transformer, so the ideal transformer model used for an iron cored transformer that electrical engineering students study in the first course on energy conversion does not apply here.

$$\begin{align*}
&v_1 \quad R_1 \quad C_1 \quad i_1 \quad \quad \quad M \quad \quad \quad R_2 \quad \quad \quad L_1 \quad \quad \quad L_2 \quad \quad \quad i_2 \quad \quad \quad v_2 \\
&\end{align*}$$

Figure 4: Lumped Circuit Model Of A Tesla Coil, arc on.

At the time the gap arcs over, all the energy is stored in $C_1$. As time increases, energy is shared among $C_1$, $L_1$, $C_2$, $L_2$, and $M$. The total energy in the circuit decreases with time because of losses in the resistances $R_1$ and $R_2$. There are four energy storage devices so a fourth order differential equation must be solved. The initial conditions are some initial voltage $v_1$, and $i_1 = i_2 = v_2 = 0$. If the arc starts again before all the energy from the previous arc has been dissipated, then the initial conditions must be changed appropriately.

The Corums present the necessary solution technique in their manual [3] and also the computer code. The voltages and currents are not single frequency sinusoids. Rather there is a frequency spectrum with one hump for $M$ small and two humps for $M$ large. This is fascinating material for lovers of circuit theory, but is of somewhat limited usefulness in suggesting design changes for better performance.

It appears to this author that the time domain solution is more useful than the frequency domain. We simply examine $v_1$, $v_2$, $i_1$, and $i_2$ as time increases, either graphically or in some sort of tabular printout. We then change one or more of the energy storage device values and do it again. It is also helpful to calculate the energy stored in each device. If the total energy stored in the circuit is decreasing monotonically with time, at the rate power is being absorbed by $R_1$ and $R_2$, then one can be reasonably confident that the computer code is working correctly.

The time domain solution resembles a drunken walk in that it is difficult to predict what a given value will do next. Energy is moving among storage devices like cannon balls rolling around on the deck of an old sailing ship. Patterns can be changed readily by changing component values. We need a strategy for evaluating each solution for movement toward or away from some optimum. This strategy is developed by recognizing the following facts. After a small number of half cycles of $i_1$, the arc will dissipate and the spark gap will again become an open circuit. At this point we want as much energy as possible stored in the secondary, either as $i^2_2 L_2/2$ or $v^2_2 C_2/2$. Any energy stored in $C_1$ when the gap opens is not available to
produce the desired high voltages on $C_2$.

With proper design (proper values of $C_1$, $L_1$, $C_2$, $L_2$, and $M$) it is possible to have all the energy in $C_1$ transferred to the secondary at some time $t_1$. That is, at $t_1$ there is no voltage across $C_1$ and no current through $L_1$. If the gap can be opened at $t_1$, then there is no way for energy to get back into the primary. No current can flow, so no energy can be stored in $L_1$, and without current the capacitor cannot be charged. The secondary then becomes a separate RLC circuit with nonzero initial conditions for both $C_2$ and $L_2$, as shown in Fig. 5. This circuit will then oscillate or “ring” at a resonant frequency determined by $C_2$ and $L_2$. With the gap open, the Tesla coil secondary is simply an RLC circuit, described in any text on circuit theory. The output voltage is a damped sinusoid.

![Figure 5: Lumped Circuit Model Of A Tesla Coil, arc off.](image)

Finding a peak value for $v_2$ given some initial value for $v_1$ thus requires a two step solution process. We first solve a fourth order differential equation to find $i_2$ and $v_2$ as a function of time. At some time $t_1$ the circuit changes to the one shown in Fig. 5, which is described by a second order differential equation. The initial conditions are the values of $i_2$ and $v_2$ determined from the previous solution at time $t_1$. The resulting solution then gives the desired peak values for voltage and current. The process is tedious, but can readily be done on a computer. It yields some good insights as to the effects of parameter variation. It helps establish a benchmark for optimum performance and also helps identify parameter values that are at least of the correct order of magnitude. However, there are several limitations to the process which must be kept in mind.

First, as we have mentioned, the arc is very difficult to characterize accurately in this model. The equivalent $R_1$ will change, perhaps by an order of magnitude, with factors like $i_1$, ambient humidity, and the condition, geometry, and temperature of the electrode materials. This introduces a very significant error into the results.

Second, the arc is not readily turned off at a precise instant of time. The space between electrodes must be cleared of the hot conducting plasma (the current carrying ions and electrons) before the spark gap can return to its open circuit mode. Otherwise, when energy starts to bounce back from the secondary, a voltage will appear across the spark gap, and
current will start to flow again, after the optimum time $t_1$ has passed. With fixed electrodes, the plasma is dissipated by thermal and chemical processes that require tens of microseconds to function. When we consider that the optimum $t_1$ may be 2 $\mu$s, a problem is obvious. This dissipation time can be decreased significantly by putting a fan on the electrodes to blow the plasma away. This also has the benefit of cooling the electrodes. For more powerful systems, however, the most common method is a rotating spark gap. A circular disc with several electrodes mounted on it is driven by a motor. An arc is established when a moving electrode passes by a stationary electrode, but the arc is immediately stretched out by the movement of the disc. During the time around a current zero, the resistance of the arc can increase to where the arc cannot be reestablished by the following increase in voltage.

The rotary spark gap still has limitations on the minimum arc time. Suppose we consider a disc with a radius of 0.2 m and a rotational speed of 400 rad/sec (slightly above 3600 rpm). The edge of the disc is moving at a linear velocity of $r\omega = 80$ m/s. Suppose also that an arc cannot be sustained with arc lengths above 2 cm. It requires $0.02/80 = 25$ $\mu$s for the disc to turn this distance. This time can be shortened by making the disc larger or by turning it at a higher rate of speed, but in both cases we worry about the stress limits of the disc. Nobody wants fragments of a failed disc flying around the room. The practical lower limit of arc length seems to be about 10 $\mu$s. With larger coils this may be reasonably close to the optimum value.

The third reason for concern about the above calculations is that the Tesla coil secondary has features that cannot be precisely modeled by a lumped circuit. One such feature is ringing at ‘harmonic’ frequencies. Neither the distributed or lumped models do a particularly good job of predicting these frequencies. Data will be presented later for a medium sized secondary (operated as an extra coil, explained in the next section), with a high Q resonance at about 160 kHz. When applied power is switched off, the coil usually rings down at 160 kHz. Sometimes, however, it will ring down at $3.5(160) = 560$ kHz. A third harmonic appears in many electrical circuits and has plausible explanations. A 3.5 ‘harmonic’ is another story entirely.

These three reasons explain why we never see a paper giving a complete Tesla coil design with experimental data verifying the theoretical design. We get started with theory, but at some point have to move to an experimental optimization. The saying is, “Tune for most smoke”, which Harry Goldman attributes to Bill Wysock and Gary Legel. It is a tribute to the experimentalists that we have coils in existence with names like “Nemesis” that can produce sparks fifteen feet long [11].

# Magnifier

As mentioned above, the classical Tesla coil uses two stages of voltage increase. Some coilers get a third stage of voltage increase by adding a magnifier coil, also called an extra coil, to their classical Tesla coil. This is illustrated in Fig. 6.
The extra coil and the air core transformer are not magnetically coupled. The output (top) of the classical coil is electrically connected to the input (bottom) of the extra coil with a section of copper water pipe of large enough diameter that corona is not a major problem. A separation of 2 or 3 meters is typical.

Voltage increase on the extra coil is by transmission line action, rather than the transformer action of the iron core transformer. Voltage increase on the air core transformer is partly by transformer action and partly by transmission line action. When optimized for extra coil operation, the air core transformer looks more like a transformer (greater coupling, shorter secondary) than when optimized for classical Tesla coil operation.

The lumped circuit enthusiast would say that voltage rise is by RLC resonance. Both camps agree that voltage rise in the secondary and especially in the extra coil are not by transformer action.

Although not shown in Fig. 6 the extra coil depends on ground for the return path of current flow. The capacitance from each turn of the extra coil and from the top terminal to ground is necessary for operation. Impedance matching from the Tesla coil secondary to the extra coil is necessary for proper operation. If the extra coil were fabricated with the same size coil form and wire size as the secondary, the secondary and extra coil tend to operate as a long secondary, probably with inferior performance to that of the secondary alone. There are guidelines for making the coil diameters and wire sizes different for the two coils, but optimization seems to require a significant amount of trial and error.

In my quest for a better description of Tesla coil operation, I decided that the extra coil was the appropriate place to start. It looks like a vertical antenna above a ground plane, so there is some prior art to draw from. While the classical Tesla coil makes an excellent driver to produce long sparks, it is not very good for instrumentation and measurement purposes. There are just too many variables. The spark gap may be the best high voltage switch available today, but inability to start and stop on command, plus heating effects, make it difficult to use when collecting data.
I therefore decided to build a solid state driver. Vacuum tube drivers have been used for many years and several researchers have developed drivers using power MOSFETs, so this was not entirely new territory. It turned out to be a long term project. At the beginning, I had little idea about the input impedance of a coil above a ground plane, or how much power would be required to get significant sparks (say, half a meter in length or more). There have been many iterations, but I finally produced a design that would make sparks. Two major disadvantages are that it requires a digital oscilloscope with deep memory for tuning purposes, and one can make longer sparks using a standard spark gap. These disadvantages make it unlikely to sweep the Tesla coil community. There might be situations, however, where this approach would be useful. One is a museum installation, for example, where sparks of 0.5 to 1 meter are acceptable, and long life and low maintenance are critical factors.

The remainder of this document is a collection of my notes on this project, including some deadends. There are discussions on

1. Capacitance
2. Inductance and Transformers
3. Gate Driver and Inverter
4. Lumped Model
5. Experimental Results

Capacitance appears in many different places in the Tesla coil system, in the power supply, the controller, the driver, the coil body itself, and the top toroid or sphere. It therefore gets a lengthy treatment. Other items get a somewhat lesser treatment.

References


