Crest Factor Considerations in MC-CDMA with Carrier Interferometry Codes

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Abstract

Carrier interferometry codes, applied to N-carrier MC-CDMA systems, enable $2N$ users to simultaneously share the system bandwidth with minimal degradation in performance (relative to the N orthogonal user case). This paper presents an analysis of crest factor considerations in MC-CDMA systems employing carrier interferometry codes. First, it is shown that the crest factor in downlink transmission demonstrates the desirable properties of low mean and low variance. Next, the poor crest factor observed in the uplink is characterized and it is shown how this can be effectively combated by use of Schroeder's analytical crest factor reduction techniques.

I. Introduction

Multi-carrier code division multiple access (MC-CDMA)[1] has emerged as a promising alternative to conventional direct sequence CDMA (DS-CDMA) in mobile wireless communications. In MC-CDMA, each user's data symbol is transmitted simultaneously over N narrowband subcarriers, with each subcarrier encoded with a -1 or $+1$ (as determined by an assigned spreading code). Multiple users are assigned unique, orthogonal (or pseudo-orthogonal) codes, allowing users to occupy identical carriers at the same time with little to no interference. Specifically, in an N-carrier MC-CDMA system, N orthogonal users can be supported simultaneously.

Recently, in [2][3], a novel code set based on carrier interferometry was proposed for MC-CDMA. Here, the spreading code corresponds to linearly increasing complex phase offsets. These carrier interferometry (CI) codes demonstrate a number of promising features. Unlike orthogonal Hadamard Walsh codes, orthogonal CI codes can be constructed with any length N. Furthermore, MC-CDMA with CI codes of length N, can support N orthogonal users as well as N additional pseudo-orthogonal users. Thus CI codes provide flexibility in design as well increased capacity, making it a strong candidate for next generation wireless systems.

One concern regarding the use of MC-CDMA with CI codes is an increased peak-to-average power ratio (PAPR) as well as increased signal dynamic range relative to single carrier schemes. This concern arises because, in the time domain, these codes create a periodic mainlobe with sidelobe activity at intermediate times. The correspondingly large PAPR leads to a reduced efficiency of the power amplifier, and an increased signal dynamic range requires power amplifiers with higher range of linearity.

In this paper, we characterize the PAPR and signal dynamic range in the down and uplink of CI/MC-CDMA systems. It is shown that the downlink of CI/MC-CDMA does not suffer from high PAPR while the uplink experiences a very high PAPR. We propose techniques to reduce PAPR and signal dynamic range in the uplink, and demonstrate that CI/MC-CDMA can be constructed with desirable PAPR and signal compactness. Specifically, the PAPR of the uplink CI/MC-CDMA is reduced to values close to that of a single sine wave.

The paper is organized as follows: Section II introduces the CI codes and corresponding MC-CDMA signaling. Section III discusses the terms used to characterize signal compactness in multi-carrier systems and provides the crest factor results for uplink and downlink CI/MC-CDMA. Section IV provides a technique to reduce crest factor in CI/MC-CDMA uplink. Finally, the conclusions are presented in Section IV.

II. CI/MC-CDMA Signaling

In MC-CDMA, the transmitted signal for user k corresponds to

$$s_k(t) = [a_k[n]p(t - nT_b)] \cdot c_k(t). \qquad (1)$$

Here, $a_k[n]$ is user k's data symbol and is assumed to be binary antipodal (n denotes the n^{th} bit interval); $p(t)$ is defined to be a rectangular pulse of unity height in the interval 0 to T_b; and $c_k(t)$ is user k's spreading code, i.e.,

$$c_k(t) = \Re e\{\sum_{i=0}^{N-1} \beta_i^k e^{j2\pi i\Delta ft}\}. \qquad (2)$$

Here, Δf is selected such that the carriers are orthogonal, i.e., $\Delta f = 1/T_b$ (typically); and, $\{\beta_i^k, i = 0, 1, ..., (N-1)\}$ is the k^{th} user's spreading code (length N). Typically, e.g., for Hadamard Walsh codes or Gold codes, $\beta_i^k \in \{-1, +1\}$. However, when CI codes are used, β_i^k's are complex phasors corresponding to

$$\{\beta_0^k, \beta_1^k, \cdots, \beta_{(N-1)}^k\} = \{1, e^{j\Delta\theta_k}, e^{j2\Delta\theta_k}, \cdots, e^{j(N-1)\Delta\theta_k}\}. \quad (3)$$

In the MC-CDMA downlink all K user's signals are bundled together prior to transmission, leading to

$$s_{down}(t) = \sum_{k=1}^{K} s_k(t); \quad (4)$$

whereas, in the uplink, each user's signal is transmitted independantly, i.e.,

$$s_{up}(t) = s_k(t); \quad (5)$$

In [2][3], it is shown that N users are supported orthogonally by selecting, for user k,

$$\Delta\theta_k = \frac{2\pi}{N}k, k = 0, 1, \cdots, N-1 \quad (6)$$

and an additional N users can be supported by also selecting

$$\Delta\theta_k = \frac{2\pi}{N}(k-N) + \frac{\pi}{N}, k = N, N+1, \cdots, 2N-1. \quad (7)$$

In this way, $2N$ users are supported on codes of length N with minimal performance degradation as the number of users increase from N to $2N$.

Viewed in the time domain, each CI code set (equation (3)) creates a code $c_k(t)$ that demonstrates periodic mainlobe with sidelobe activity at intermediate times (Figure 1). From this figure, it is apparent that the crest factor (defined in section III) in the uplink will be very high.

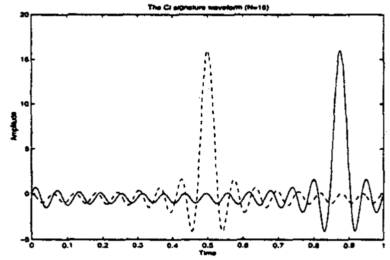

Figure 1: Two CI signature waveforms: user k's (solid line) and user j's (dotted line)

III. Crest Factor Issues

To measure the signal compactness of the CI/MC-CDMA signal, we employ the crest factor (CF). For a multi-carrier signal $u(t)$ [5]

$$CF = \frac{M^+ - M^-}{2E_{eff}} \quad (8)$$

where M^+ is the largest positive and M^- is the most negative value of $u(t)$. E_{eff} represents the total amount of energy contained in $u(t)$ and equals $\|u\|_2$, i.e., the rms value of $u(t)$. It is clear from (1) that a sine wave has a crest factor of $\sqrt{2}$. Another common approximation found in the literature [6] relates crest factor to PAPR:

$$CF \approx \sqrt{PAPR} = \frac{\|u\|_\infty}{\|u\|_2} \quad (9)$$

where $\|u\|_\infty$ corresponds to the maximum absolute value of $u(t)$. In the following discussion, we provide exact CF values in accordance with (8) as well as $\sqrt{PAPR}$ (see (9)) values for CI/MC-CDMA.

A. Downlink CF

The crest factor of $s_{down}(t)$ is dependent on the binary antipodal data symbols of the users, $a_k[n]$, equally likely to be +1 or -1. The crest factor is therefore a Rayleigh distributed random variable [7][8]. Assuming $N = 32$ carriers and $K = 32$ users, the probability density functions of CF and $\sqrt{PAPR}$ for $s_{down}(t)$ are plotted in Figures 2 and 3 respectively. These results were obtained using computer-based simulation of the transmitted signal $s_{down}(t)$ with random binary data, followed by evaluation of the CF values based on (8) and (9). It is observed that $E[CF] = 1.85$ and $E[\sqrt{PAPR}] = 1.89$, where $E[\cdot]$ refers to the numerical mean, which is well within tolerable levels of power amplifiers. The probability of $CF \geq 2.05$ is less than 4%, indicating that the variance of CF is low, i.e., CF rarely ventures far above its mean of 1.85. This low CF can be attributed to simultaneous transmission of all users' signals. That is, even though the signature waveform of Figure 1 appears to have a poor CF, the combined signal, with all signature waveforms and data symbols on them, actually improves the CF tremendously. An example of this phenomenon is shown in Figure 4, where, when bits $a_k[n]$ are '1' for all users $k \in \{0, 1, ..., N-1\}$, the combined signal (solid line) is seen to demonstrate smaller values of PAPR and CF.

B. Uplink CF

We now turn our attention to the CF in the CI/MC-CDMA uplink. The crest factor of $s_{up}(t)$ is shown in Table 1 where it is apparent that CF and $\sqrt{PAPR}$

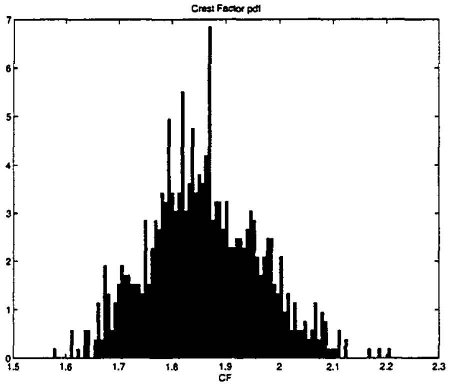

Figure 2: Probability density function of CF (N=32, K=32)

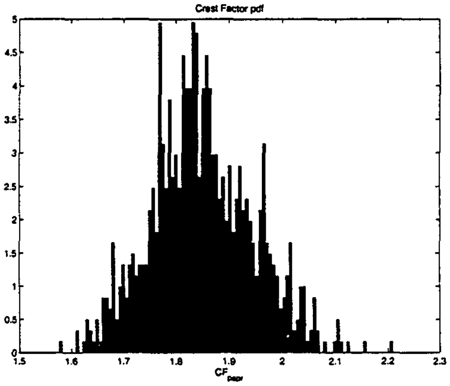

Figure 3: Probability density function of $\sqrt{PAPR}$ (N=32, K=32)

increases with increasing N. These values were calculated analytically, since the uplink CI/MC-CDMA signal envelope has a fixed maximum, minimum and rms value that does not depend on the transmitted data.

Systems built to accomodate such crest factor values would be extremely inefficient as high CF would cause the power amplifier to 'back-off' (reduce average transmission power) to avoid non-linear distortions. Hence, we employ a CF reduction technique that enables the CI/MC-CDMA system to demonstrate desirable CF values in the uplink.

N	CF	$\sqrt{PAPR}$
8	2.59	4.00
16	3.55	4.94
32	4.95	8.00

Table 1: CI/MC-CDMA Uplink CF values

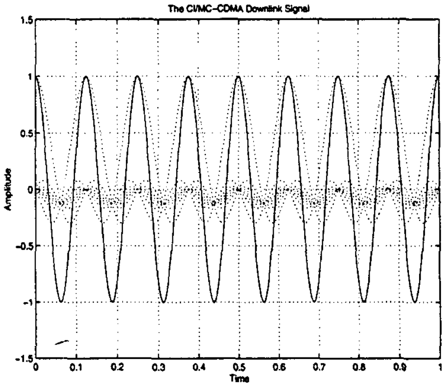

Figure 4: Signal Envelope of CI/MC-CDMA downlink ($a_k[n] = 1$ for all users)

IV. CF Reduction Technique

In a multi-carrier signal, the crest factor is a function of the phase angles of the carriers. Schroeder [9] proposed a powerful, easy-to-implement rule for phase angle adjustment which was shown to be effective in cases where the carriers are concentrated in a small frequency band (small relative to the center frequency). MC-CDMA falls under this category and hence in this section we apply Schroeder's technique to reduce CF in the CI/MC-CDMA uplink. (It is important to note that there are other techniques of phase angle adjustments ([10],[11]) that can alternatively be employed in MC-CDMA systems to reduce CF).

Applying [9], the CF is reduced in the uplink of CI/MC-CDMA by introducing a phase offset into each carrier at the transmitter side, i.e., the spreading code for the uplink of CI/MC-CDMA is updated from (3) and now corresponds to

$$(\beta_0^k, \beta_1^k, \cdots, \beta_{(N-1)}^k) = (e^{j\phi_0}, e^{j\Delta\theta_k + j\phi_1}, \cdots, e^{j(N-1)\Delta\theta_k + j\phi_{(N-1)}}), \quad (10)$$

where $\{\phi_0, \phi_1, ..., \phi_{N-1}\}$ are determined as follows. We start with a random phase for the first carrier, ϕ_0, and calculate the phases for the $N-1$ remaining carriers $\{\phi_1, \cdots, \phi_{N-1}\}$ using [9]

$$\phi_n = \phi_0 - \frac{\pi n^2}{N}, n = 1, 2, ..., (N-1). \quad (11)$$

The CF for the signal (with these phase offsets) is computed and stored. Starting with a new random value for ϕ_0, we repeat equation (11) to determine a new $\{\phi_0, \phi_1, ..., \phi_{N-1}\}$. The CF for this signal with the new phase offsets are again computed. This procedure is repeated approximately 1000 times and the

CF values computed are inspected to determine the $\{\phi_0, \phi_1, \cdots, \phi_{N-1}\}$ that minimized CF.

Alternately, in [9], a second method is provided where we restrict ϕ_i such that $\phi_i \in \{0, \pi\}$. Here, the phase offsets are computed using

$$\phi_n = \pi \lfloor \frac{n^2}{2N} \rfloor, n = 0, 1, ..., (N-1) \tag{12}$$

where $\lfloor x \rfloor$ indicates the largest integer not larger than x. The results of applying both these methods are presented in Table 2 and Table 3. These results were obtained for a CI/MC-CDMA system uplink employing N=8,16 and 32 carriers.

From Table 2, we observe (using Shroeder's method to determine $\{\phi_0, \phi_1, \cdots, \phi_{N-1}\}$) CF values are reduced from 4.9 to 1.6 when N=32. Even after limiting the phase offsets ϕ_i such that $\phi_i \in \{0, \pi\}$, we observe an improvement from 4.9 to 1.8. Significant improvements are also seen in $\sqrt{PAPR}$ values. Specifically, the CF of the uplink is now reduced to values close to that of a single sine wave (CF of sine wave = 1.414). Hence, the CF problem of uplink CI/MC-CDMA is efficiently solved.

Finally, it is also important to note that the signature waveforms $c_k(t)$ and $c_j(t)$ in a CI/MC-CDMA system are simply time shifted versions of one other (Figure 1). Hence, it suffices to calculate one set of phase offsets $\{\phi_0, \phi_1, \cdots, \phi_{N-1}\}$ to reduce the CF and this set can be applied to all the users' spreading codes. (In MC-CDMA systems applying e.g., Gold codes or Hadamard Walsh codes, the same phase offsets cannot be used for all users' spreading codes).

V. Conclusions

In summary, this paper addresses the problem of signal compactness in CI/MC-CDMA as measured by crest factor. An analysis of the CI/MC-CDMA downlink shows that crest factor is well within tolerable levels of power amplifiers. On the other hand, the CI/MC-CDMA uplink suffers from high CF values. However, applying Schroeder's CF reduction technique, we demonstrate that the uplink CF can be brought to very low values (close to that of a pure sine wave). Hence, we demonstrate that the CI/MC-CDMA system not only serves as a flexible, high capacity CDMA system, but, it is also available with very low PAPRs (i.e., it makes very efficient use of power amplifiers).

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N	Method I CF	Method I $\sqrt{PAPR}$	Method II CF	Method II $\sqrt{PAPR}$
8	1.66	1.69	1.67	1.86
16	1.71	1.73	1.88	1.97
32	1.66	1.67	1.85	1.85

Table 2: Schroeder's methods to reduce uplink CF (Method I (any phase); Method II (0 or π))

ϕ	Method I (in rad.)	Method II (in rad.)
0	0.2899	0
1	-1.2809	0
2	-3.2444	0
3	-5.9933	π
4	-3.2444	π
5	-1.2809	0
6	-0.1028	π
7	-5.9933	0

Table 3: ϕ_n values minimizing CF in CI/MC-CDMA uplink (determined from Schroeder's method) for $N = 8$ carriers.