This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

Limited Feedback-based Unitary Precoder Design in Rician MIMO Channel via Trellis Exploration Algorithm

Dalin Zhu
Department of Wireless Communications
NEC Laboratories China (NLC)
11F Bld.A Innovation Plaza Thus Park, Beijing, China 100084
E-mail: zhu_dalin@nec.cn

Balasubramaniam Natarajan
Department of Electrical & Computer Engineering
Kansas State University, Manhattan, KS 66506
E-mail: bala@ksu.edu

***Abstract*—It is well known that in practice, MIMO channels usually exhibit strong line-of-sight (LoS) components. However, common codebook designs in precoded MIMO rely heavily on the assumption of i.i.d Rayleigh fading. Therefore, designing practical unitary precoders that are suited for realistic channel condition (such as Rician MIMO channel) becomes an interesting topic to investigate. In this paper, we propose to use trellis exploration algorithm to accomplish this objective by adapting each entry of the precoding matrix to instantaneous channel realizations. Since trellis search is an on-line learning approach, our proposed precoding scheme is more adaptive to variations in channel propagations than off-line designed codebooks. Specifically, we use simulation results to show that in Rician MIMO channel, our proposed approach results in better performance than Grassmannian line/space packing and typical DFT codebook design in terms of ergodic capacity, under the same amount of feedback bits.**

I. Introduction

Multiple-input multiple-output (MIMO) is a promising technique for next generation wireless communication systems. Since both the transmitter and receiver are equipped with multiple antennas, MIMO systems can support high data rates and link reliability through multiplexing and diversity gain, respectively [1]. Precoding is widely used in MIMO for maximizing system throughput and minimizing error probability. Precoding schemes that typically require channel state information (CSI) at the transmitter are implemented by feeding a large amount of quantized channel fade parameters from the receiver back to the transmitter through a delay and error free reverse channel. However, it becomes more challenging when the reverse channel is a limited-rate feedback channel. To overcome this problem, limited feedback-based codebook design is proposed. In limited feedback-based precoding scheme, the receiver first computes the optimal precoder from a finite set of candidate vectors/matrices (also known as codebook). Then, the index of the optimal precoder in that finite space is fed back to the transmitter as representative quantized CSI [2].

Vector quantization (VQ)-based codebook design has been extensively investigated in [3][4][5]. Generally, Lloyd algorithm is invoked in VQ-based codebook design to asymptotically reduce the capacity loss with increase in the codebook size. One another popular codebook design is the so-called Grassmannian line/space packing (GLP/GSP) [6][7]. In general, the goal of GLP/GSP is to find a set of subspaces such that their subspace distances are maximized. However, GLP/GSP design relies heavily on the assumption of i.i.d (independently identically distributed) Rayleigh fading channel. It may be difficult to find good GLP/GSP codebooks that are suited for MIMO channel with strong LoS components. DFT codebook is being considered as the standardizing codebook in 3GPP LTE [8]. DFT codebook has constant modulus structure which is suited for strong LoS environment from the standpoint of power balance. Extensive research effort on low complexity DFT codebook design can be found in [9]. However, typical codebooks are designed off-line, which are not adaptive enough to variations in transmission environment. Therefore, there is a need for limited feedback-based unitary precoder design approach such that the resulting precoders are highly adaptive to various propagation conditions.

In this paper, we propose a novel on-line learning approach termed as trellis exploration algorithm to design practical unitary precoding matrices. Trellis exploration algorithm is a well-known dynamic programming approach that was previously used as a tool for determining optimal signature sequences in DS-CDMA systems [10]. Specifically, in this paper, the unitary precoder is constructed as the product of one arbitrary DFT matrix (or one arbitrary matrix with orthonormal columns) and one modifying diagonal matrix. The polyphase modifying diagonal matrix is custom designed via the trellis exploration algorithm employing MIMO capacity as the metric to maximize. Since the trellis exploration algorithm only operates on a finite set of phases, the unitary precoding matrix is first determined at the receiver; then a small number of bits indicating the corresponding phase indices are fed back to the transmitter as representative quantized CSI. In contrast to typical codebook design, trellis search-based method adapts each entry of the precoding matrix to instantaneous channel realizations. In this way, it exhibits high adaptivity to various channel conditions. In addition, trellis search-based precoding matrix also has constant modulus structure (as compared to

DFT codebook design), which is suited for practical MIMO channel with strong LoS components from the standpoint of power balance.

The rest of the paper is organized as follows: In section II, system model along with analysis on optimal precoder design in Rician MIMO channel is given. Our proposed trellis approach is illustrated in section III. In section IV, simulation results are presented. Finally, we conclude this paper in section V.

II. System Model

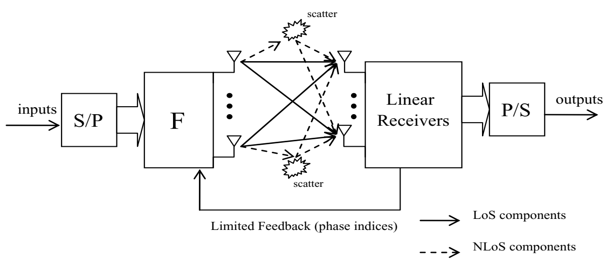

Fig. 1. Block diagram of a closed-loop precoded MIMO system

The block diagram of a closed-loop precoded MIMO system with M_t transmit antennas and M_r receive antennas is given in Fig.1. Binary inputs are first modulated and then grouped into vectors with length M symbols each, i.e., $\mathbf{x} = (x_1, x_2, \cdots, x_M)^T$, where $(\cdot)^T$ denotes transposition operation and $M < M_t$. The covariance matrix $\mathbf{\Sigma}$ of the transmitted symbol vector is $\mathbf{\Sigma} = E(\mathbf{x}\mathbf{x}^H)$, where $(\cdot)^H$ denotes conjugate transposition and $E(\cdot)$ is the expectation operation. Therefore, if the total transmit power is P_t, tr$(\mathbf{\Sigma}) \leq P_t$ must hold, where tr$(\cdot)$ represents matrix trace operation. The transmitted symbol vector $\mathbf{x}$ is then multiplied by a $M_t \times M$ precoding matrix $\mathbf{F}$, resulting in a length M_t data vector $\mathbf{s}$ (i.e., $\mathbf{s} = \mathbf{F}\mathbf{x}$), where $\mathbf{F} \in U(M_t, M)$, the set of $M_t \times M$ complex matrices with orthonormal columns. Then, $\mathbf{s}$ is transmitted via M_t transmit antennas over a wireless environment. The received vector $\mathbf{y} = (y_1, y_2, \cdots, y_{M_r})^T$, corresponds to

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}, \tag{1}$$

where, $\mathbf{n}$ is a $M_r \times 1$ additive cmplex Gaussian noise vector with each entry distributed according to $\mathcal{CN}(0, N_0)$. $\mathbf{H}$ represents the $M_r \times M_t$ MIMO channel matrix, which in general, corresponds to

$$\mathbf{H} = \sqrt{\frac{K}{K+1}} \cdot \mathbf{H}_{LoS} + \sqrt{\frac{1}{K+1}} \cdot \mathbf{H}_{NLoS}. \tag{2}$$

Here, K is the Ricean factor [11]; $\mathbf{H}_{NLoS}$ is the $M_r \times M_t$ non-line-of-sight (NLoS) channel matrix whose entries are distributed according to $\mathcal{CN}(0, 1)$ that captures reflections, diffractions and scattering from the wireless environment; $\mathbf{H}_{LoS}$ is the $M_r \times M_t$ LoS channel matrix that characterizes the free-space components between transmit and receive antennas. Specifically, $\mathbf{H}_{LoS}$ can be expressed as

$$\mathbf{H}_{LoS} = [\mathbf{h}_0, \mathbf{h}_1, \cdots, \mathbf{h}_{M_t-1}]. \tag{3}$$

Here, $\mathbf{h}_n$ corresponds to,

$$\mathbf{h}_n = \left[\exp\left(\frac{j2\pi}{\lambda} r_{0,n}\right), \cdots, \exp\left(\frac{j2\pi}{\lambda} r_{M_r-1,n}\right)\right]^T, \tag{4}$$

where, λ is the wavelength of the carrier, $r_{m,n}$ denotes the path length between transmit antenna n and receive antenna m, actually characterizing the angle of departure (AoD) and angle of arrival (AoA) in a LoS MIMO channel [12], expressed as

$$r_{m,n} = \|\mathbf{a}_m - \mathbf{a}_n\|_2^2. \tag{5}$$

$\mathbf{a}_n$ and $\mathbf{a}_m$ are vectors from origo[1] to the transmit antenna n, and from origo to the receive antenna m, respectively, in a ray tracing LoS channel model [12].

In this paper, we mainly focus on maximizing MIMO capacity. Assuming equal power allocation, i.e., $\mathbf{\Sigma} = \frac{P_t}{M_t}\mathbf{I}_{M_t}$, the capacity of MIMO channel is calculated as [13],

$$C = \log_2 \det\left(\mathbf{I}_{M_r} + \frac{P_t}{M_t N_0}\mathbf{H}^H\mathbf{H}\right), \tag{6}$$

where, $\mathbf{I}_{M_r}$ is the $M_r \times M_r$ identity matrix, $\det(\cdot)$ is the matrix determinant operation. Therefore, for a precoded MIMO with $M_t \times M$ precoding matrix $\mathbf{F}$, we can rewrite (6) as

$$C = \log_2 \det\left(\mathbf{I}_M + \frac{P_t}{M N_0}\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F}\right). \tag{7}$$

Hence, the optimization problem is formulated as

$$\mathbf{F}_{opt} = \arg \max_{\mathbf{F} \in U(M_t, M)} \log_2 \det\left(\mathbf{I}_M + \frac{P_t}{M N_0}\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F}\right). \tag{8}$$

From (8), we can see that in order to maximize the system throughput, we need to maximize the effective channel ($\mathbf{HF}$) power gain. We consider two extreme scenarios, which are NLoS MIMO ($K = 0$) and LoS MIMO ($K = \infty$). For NLoS MIMO (i.e., i.i.d Rayleigh fading), the problem of designing optimal unitary precoders is equivalent to [6]. That is, precoding matrices are determined to minimize the upper bound of the average distortion function (ADF), where the upper bound of the ADF is obtained by averaging over i.i.d Rayleigh fading channel components [6]. Since the upper bound of the ADF is a function of subspace distances, the codebook design criterion is to find a set of unitary vectors/matrices such that their subspace distances are maximized [6][7]. However, (1) typical codebook is designed off-line, which is not adaptive enough to variations in transmission environment; (2) the upper bound of the ADF derived in [6] is not tight [14], which in turn, may result in difficulties in finding good codebooks even for i.i.d Rayleigh fading channel.

For LoS MIMO, channel components are not i.i.d random variables any more, but are fixed phase values with constant amplitudes. For simplicity of analysis, we assume $M = 1$ (i.e.,

[1] origo is defined as the lower end of the transmit array. Geometrical illustration can be addressed to Fig.1 in [12].

beamforming), $M_r = 1$ (i.e., MISO) and uniform linear array (ULA). Hence, (3) can be rewritten as,

$$\mathbf{h}_{LoS} = \left[\exp\left(\frac{j2\pi d}{\lambda}\cos\theta\right), \cdots, \exp\left(\frac{j2M_t\pi d}{\lambda}\cos\theta\right)\right]^T, \tag{9}$$

where d is the spacing between antennas in ULA, θ denotes the AoD and if small angle spread is assumed at the transmitter, all AoDs are approximately the same [9]. Therefore, the optimal beamformer $\mathbf{F}_{opt}$ can be obtained by

$$\mathbf{F}_{opt} = \arg\max_{\mathbf{F}\in U(M_t,1)} |\mathbf{h}_{LoS}^T\mathbf{F}|^2. \tag{10}$$

Thus, given AoD and $\mathbb{F} = \{\mathbf{F}_1, \cdots, \mathbf{F}_N\}$, $G_d[\mathbb{F}, \mathbf{h}_{LoS}|\theta]$ can be used as a metric to measure the robustness of the codebook $\mathbb{F}$, which corresponds to

$$G_d[\mathbb{F}, \mathbf{h}_{LoS}|\theta] = \max_{\mathbf{F}\in\mathbb{F}} |\mathbf{h}_{LoS}^T\mathbf{F}|^2. \tag{11}$$

Next, we illustrate why precoder that has constant modulus structure is more suited for LoS MIMO. (11) can be further upper bounded as

$$\begin{aligned} G_d[\mathbb{F}, \mathbf{h}_{LoS}|\theta] &= \max_{\mathbf{F}\in\mathbb{F}} |\mathbf{h}_{LoS}^T\mathbf{F}|^2 \\ &\leq \|\mathbf{h}_{LoS}^T\|_2^2 \qquad (12) \\ &\leq M_t. \qquad (13) \end{aligned}$$

Equality in (12) holds only when $\mathbf{F} = \mathbf{h}_{LoS}^*$. Therefore, the upper bound in (12) is more likely to be achieved if $\mathbf{F}$ is chosen from a codebook that has constant modulus inputs, rather than from GLP/GSP. Moreover, with increase in the codebook size, constant modulus codebook becomes more and more adaptive to the variations in AoDs, which indicates that the equality in (13) is more likely to be achieved under this scenario. Next, at high SNR, we approximate the capacity loss due to quantization for LoS MIMO. For simplicity, we assume maximum ratio transmission (MRT) and beamforming. The corresponding capacity loss is given as

$$\begin{aligned} C_{loss} &= \log_2\left(1 + \frac{\lambda_1 P_t}{N_0}\right) - \log_2\left(1 + \frac{|\mathbf{h}_{LoS}^T\mathbf{F}|^2}{N_0}\right) \\ &\leq \log_2\left(1 + \frac{\lambda_1 P_t}{N_0}\right) - \log_2\left(1 + \frac{\lambda_1 P_t}{N_0}|u_1^H\mathbf{F}|^2\right) \\ &\approx \log_2\lambda_1 - \log_2\left(\lambda_1|u_1^H\mathbf{F}|^2\right) \\ &= \log_2\left(\frac{1}{G_d[\mathbb{F}, \mathbf{h}_{LoS}|\theta]}\right), \qquad (14) \end{aligned}$$

where λ_1 is the largest eigenvalue of $\mathbf{h}_{LoS}$ and u_1 denotes its corresponding eigenvector. From (14), we can see that the capacity loss for LoS MIMO does not depend on the chordal distance between two subspaces any more (which is the case for NLoS MIMO), but the transmit steering vector that has constant modulus inputs.

So far, we have analyzed optimal precoder design criteria for both NLoS and LoS MIMO channels. However, in practice, the Rician factor K is generally not zero, nor infinity, but some constant value. In [15], codebook that contains both codewords that are suited for LoS MIMO (e.g., DFT matrices) and codewords that are suited for NLoS MIMO (e.g., GLP/GSP) is proposed. However, performance loss will be observed when $K \to 0$ or $K \to \infty$ as compared to typical codebook design. Therefore, there is a need for a unified unitary precoder design approach that is suited for all scenarios.

III. Proposed Approach

A. Proposed precoding scheme

In this paper, we invoke the use of trellis exploration algorithm to determine optimal precoders that maximize MIMO capacity under all scenarios. Specifically, $\mathbf{F}$ corresponds to

$$\mathbf{F} = \mathbf{PT}, \tag{15}$$

where $\mathbf{T}$ is an arbitrary $M_t \times M$ DFT matrix, and $\mathbf{P}$ is a $M_t \times M_t$ modifying diagonal matrix, corresponding to,

$$\mathbf{P} = \begin{pmatrix} p_{1,1} & 0 & \cdots & 0 \\ 0 & p_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \cdots & p_{M_t,M_t} \end{pmatrix}. \tag{16}$$

Here, $p_{i,i}$ ($i = 1, \cdots, M_t$) denotes a unique phase value that is chosen from a phase set containing L candidate phases. The candidate phases $\Theta_1, \cdots, \Theta_L$ are uniformly distributed in the discrete phase space, i.e.,

$$\Theta_l = \frac{1}{M}\exp(j2\pi\frac{l-1}{L}), l = 1, 2, \cdots, L. \tag{17}$$

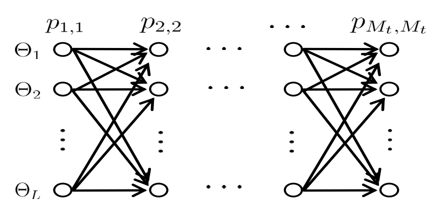

Fig. 2. Trellis diagram

The trellis diagram corresponding to the trellis exploration algorithm is given in Fig.2, which comprises of M_t state transitions, with each transition point having L nodes. A collection of branches through the trellis diagram from a beginning node to an end node is typically referred to as a path. The trellis exploration algorithm selects a path through the trellis such that nodes along the path (representing diagonal elements of matrix $\mathbf{P}$) maximize system capacity.

The operation of the proposed trellis exploration algorithm is presented next: The node at each transition point selects the winning path from one of the nodes at previous transition point by appending its relative phase and recalculating current C. Then, at current stage, phase which leads to the maximization on C is chosen as the corresponding diagonal entry of $\mathbf{P}$. It is evident that a candidate path at state $p-1$ can be selected more than once by nodes at state p, and paths with poorer metrics are discarded. Finally, nodes collected by the best path through

978-1-4244-5638-3/10/$26.00 ©2010 IEEE

the trellis diagram with maximum branch metric (i.e., C) are determined as corresponding diagonal entries of matrix $\mathbf{P}$. In our proposed algorithm, the first $M_t - 1$ diagonal elements of matrix $\mathbf{P}$ have no knowledge about what the subsequent phases will be. Hence, it is necessary to apply multiple iterations through the trellis to exploit the complete information.

Our proposed approach has several features that are important to note: (1) since the trellis exploration algorithm operates on a finite set of phases, the precoding matrix $\mathbf{P}$ is first custom designed at the receiver; then, phase indices of the corresponding $\mathbf{P}$ are fed back to the transmitter as representative quantized CSI and the phase set is shared by both the transmitter and receiver. The number of feedback bits is calculated as follows: $k_b = \lceil \log_2 L \rceil$, i.e., k_b bits are needed to specify one phase element in the phase set containing L phases. Therefore, for a $M_t \times M_t$ $\mathbf{P}$ with its diagonal entries determined by trellis search, a total of $k_b M_t$ bits are fed back to the transmitter; (2) $\mathbf{T}$ is an arbitrary DFT matrix that is mainly introduced for matrix rank adaptation. We further note that $\mathbf{T}$ can be arbitrary as the determination of each element in $\mathbf{P}$ has taken the effect of $\mathbf{T}$ into account. Therefore, once $\mathbf{T}$ is determined, it is used in all cases and also shared by both the transmitter and receiver.

Next, we illustrate why trellis approach is more adaptive to variations in transmission environment comparing with typical codebook design: (a) For NLoS MIMO, in contrast to GLP/GSP, the rank adaptation matrix in trellis approach not only adjusts the precoding matrix to a proper rank, but also introduces more degrees of freedom in designing optimal precoders under the same amount of feedback bits (As the rank adaptation matrix does not need to be transmitted from the receiver back to the transmitter). Additionally, trellis exploration algorithm is an on-line learning approach, which adapts each entry of the precoding matrix to the instantaneous channel realizations; (b) For LoS MIMO, trellis search-based precoders have constant modulus structures as well. Furthermore, in contrast to DFT codebook design, trellis approach is more adaptive and has more design degrees of freedom as described in (a); (c) From (a) and (b), it is straightforward to see that trellis approach operates on a fixed finite set of phases, which does not change with respect to K-factor. However, phases that can be selected via trellis search to construct the precoding matrix rely on K-factor, which in turn, reveals the adaptivity of our proposed approach.

B. Storage space

Storage space and computational complexity are important considerations regarding practical implementations. In this paper, storage space can be interpreted as the amount of information (in terms of bits) that needs to be stored before processing/pre-multiplying. In limited feedback-based codebook design, both transmitter and receiver sides have to store the codebook in advance. In order to achieve desired performance gain, a relatively large codebook needs to be stored in memories.

We denote n_b as the number of bits that represent one complex entry in the storage space. Therefore, for a $M_t \times M$ codeword ($M_t M$ complex entries), $n_b M_t M$ bits are required. If the codebook contains N codewords, a total of $n_b N M_t M$ bits are needed to store the entire codebook. We note that same codewords may exist in the codebook (e.g., DFT codebook). Under this scenario, storage space needed is slightly less than $n_b N M_t M$ bits. However, the worst case scenario corresponding to a total of $n_b N M_t M$ bits need to be stored.

In the proposed trellis search-based precoding scheme, one $M_t \times M$ arbitrary DFT matrix is first stored. This requires $n_b M_t M$ bits. We further note that once the DFT matrix is determined, it remains the same through the entire design process (this indicates that the number of bits used to store this DFT matrix is independent on N). Next, one $M_t \times M_t$ modifying diagonal matrix is stored, which demands $n_b M_t$ bits. Finally, the storage space of L candidate phases is calculated as $n_b L$ in terms of binary bits. Therefore, a total of $n_b(M_t M + M_t + L)$ bits storage space is needed to facilitate our proposed approach. Straightforwardly, in contrast to typical codebook design, the storage space used by the proposed precoding scheme is much less under the same amount of feedback bits. Two illustrative examples are given in Table I. In the first example, beamforming is assumed, i.e., $M = 1$; 4 transmit antennas are employed in the system with 4 feedback bits, which indicates that $N = 2^4 = 16$ and $L = 2$. The second example considers rank-2 precoder design, which implies that $M = 2$. Other parameters are the same as in the first example.

TABLE I
STORAGE SPACE COMPARISON

	$M=1$	$M=2$
Typical codebook	$64n_b$	$128n_b$
Trellis approach	$10n_b$	$14n_b$

C. Computational complexity

Low computational complexity is always of great interest to reduce the memory overhead. Especially in highly dynamic environment with high doppler spread, instantaneous adaptation to the changing environment becomes more and more critical. In this part, we quantify the computational complexity as the number of arithmetic operations that need to be computed. This essentially includes complex additions and multiplications that are performed. Again, typical codebook designs are considered as benchmarks for comparison. For simplicity, we assume that the trellis approach and typical codebook design employ the same metric, i.e., $\|\mathbf{HF}\|_2^2$, to maximize. Since the computation of matrix norm is the same for all arithmetic calculations, the search complexity is mainly subject to the computation of $\mathbf{HF}$. Straightforwardly, with $M_r \times M_t$ $\mathbf{H}$ and $M_t \times M$ $\mathbf{F}$, a total of $MM_r(M_t - 1)$ complex additions and MM_tM_r complex multiplications are performed for calculating one-shot $\mathbf{HF}$.

For typical codebook design, given $\mathbf{H}$, $\mathbf{F}_{opt}$ that maximizes

TABLE II
COMPUTATIONAL COMPLEXITY (MADS) COMPARISON: BEAMFORMING (M=1)

	4 bits	8 bits
Typical codebook	448	7168
Trellis approach	896	1792

TABLE III
SYSTEM PARAMETERS

Number of Tx antennas	4
Number of Rx antennas	4
Rank M	1 or 2
Number of feedback bits	4 or 8
Modulation scheme	QPSK
Number of phases L	2 or 4
Number of iterations I	4
Channel model	Rician MIMO

$\|\mathbf{HF}\|_2^2$ is exhaustively searched through N codewords. Therefore, in this case, finding optimal unitary precoder requires $NMM_r(M_t-1)$ complex additions and NMM_tM_r complex multiplications.

For our proposed trellis approach, the determination of one diagonal entry of $M_t \times M_t$ $\mathbf{P}$ involves exhaustive search through L candidate phases. Therefore, one-shot exploration through the trellis diagram from the beginning node to the end node requires a total of $LMM_rM_t(M_t-1)$ complex additions and $LMM_rM_t^2$ complex multiplications. As mentioned above, one-shot trellis search may not exploit the complete information. Therefore, if I iterations through the trellis is performed, the computational complexity becomes $ILMM_rM_t(M_t-1)$ and $ILMM_rM_t^2$ in terms of complex additions and multiplications, respectively.

Two examples illustrating the comparisons between the trellis approach and typical codebook design in terms of computational complexity are given in Table II. Under this scenario, both 4 feedback bits and 8 feedback bits are taken into account, which correspond to $N = 2^4 = 16, L = 2$ and $N = 2^8 = 256, L = 4$, respectively. Other parameters are given as follows: $M_t = M_r = 4, I = 4$. From Table II, we can see that with 4 feedback bits, trellis approach shows a moderate increase in computational complexity relative to typical codebook design. However, with increase in the number of feedback bits, computational complexity can be significantly reduced by trellis search in contrast to typical codebook design.

IV. SIMULATION RESULTS

In this section, we evaluate using simulations, the performance of our proposed trellis exploration algorithm aided unitary precoder design with limited feedback. Parameters of the simulated system are given in Table III.

The measurement of codebook robustness G_d is plotted in Fig.3 for various precoding schemes. It is observed that the trellis approach achieves the largest G_d for almost all

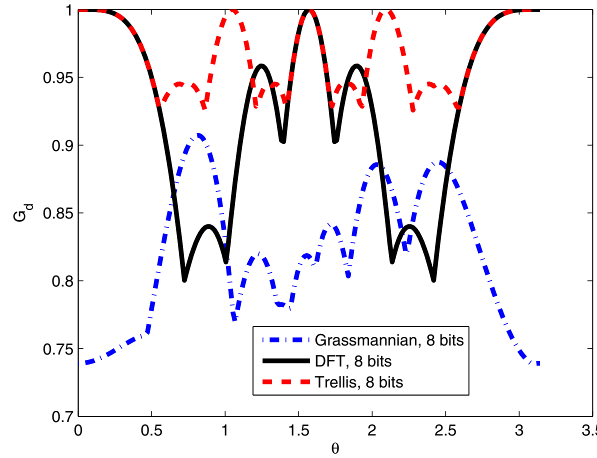

Fig. 3. G_d of various precoding schemes in LoS MIMO with $d/\lambda = 0.5$. 4×4 precoded LoS MIMO with 4×1 $\mathbf{F}$ is evaluated with 8 feedback bits

the AoDs. From Fig.3, we also observe that typical DFT codebook design is inferior relative to the trellis approach but generally better than GLP in terms of G_d. This is because although DFT codebook has constant modulus structure, its smaller alphabet size (in contrast to trellis search) limits its capability of exploiting suboptimal solutions. Furthermore, Grammannian line packing shows the worst case scenario among the three as its construction is built on the assumption of i.i.d Rayleigh fading.

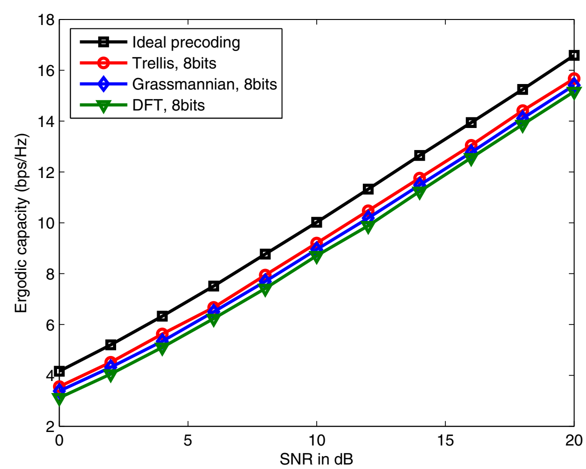

Fig. 4. Ergodic capacity of 4×4 precoded NLoS MIMO with 4×2 $\mathbf{F}$.

In Fig.4, ergodic capacities of precoded MIMO with GSP, DFT codebook and trellis exploration algorithm assisted precoding schemes are evaluated. Here, the ergodic capacity is defined as $C_{ergodic} = E_H(C)$, where the expectation is taken with respect to instantaneous channel realizations. It is observed that with 8 feedback bits ($L = 4$), trellis approach shows better performance than GSP and DFT codebook in terms of ergodic capacity. This is expected as the trellis approach introduces more design degrees of freedom relative to GSP and DFT codebook designs. Furthermore, trellis approach is more adaptive to instantaneous channel realizations than other off-line designed codebooks.

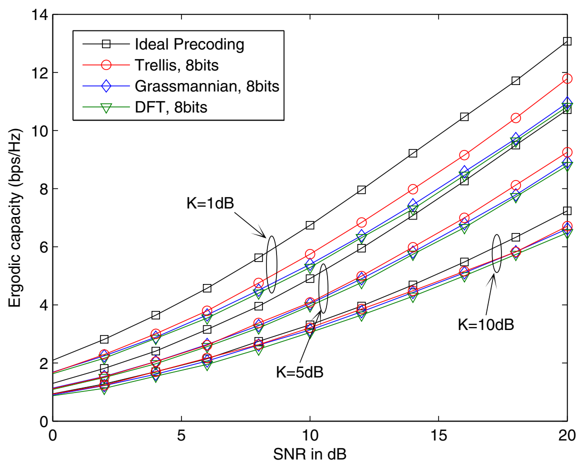

Fig. 5. Ergodic capacity of 4×4 precoded Rician MIMO with 4×2 $\mathbf{F}$.

In this example, the Ricean K factor is given as some constant values characterizing both NLoS and LoS components in the environment. The corresponding simulation result is shown in Fig.5. Here, K is set to be 1dB, 5dB and 10dB, respectively. From Fig.5, it is observed that for different K values, our proposed trellis search aided precoding scheme still shows superior performance over GSP and DFT codebook design, in terms of ergodic capacity.

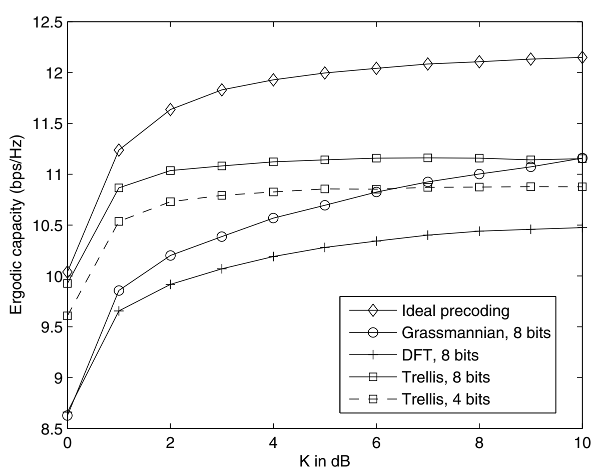

Fig. 6. Ergodic capacity versus Rician factor K of 4×4 precoded Rician MIMO with 4×2 $\mathbf{F}$.

In Fig.6, system ergodic capacity versus Rician K-factor is plotted using GSP, DFT codebook and trellis approach, in a 4×4 MIMO. We first observe that under the same amount of feedback bits (i.e., 8 in this example), trellis approach is more adaptive to the variations in K-factor, as compared to GSP and DFT codebook design. However, this gain decreases with increase in K-factor, and this observation is consistent with the results shown in Fig.5. In addition, we present trellis approach with 4 feedback bits in this example as well. We can see that our proposed approach with 4 feedback bits exhibits even better capacity performance than GSP with 8 feedback bits, if Rician factor K is less than, say, 6.3dB. Therefore, under this scenario, one can switch between trellis approach and GSP regarding practical concerns.

V. Conclusion

In this paper, we present a novel methodology (i.e., trellis exploration algorithm) to design optimal unitary precoders in Rician MIMO channel. Since the resulting precoding matrices have constant modulus and potentially more design degrees of freedom, our proposed approach outperforms typical off-line designed codebooks in terms of ergodic capacity, under the same amount of feedback bits.

References

[1] A. Paulraj, D. A. Gore, and H. Bolcskei, "An overview of MIMO communication - a key to gigabit wireless," in *Proceedings of the IEEE*, vol. 92, no. 2, Feb. 2004, pp. 198–218.

[2] D. J. Love, R. W. Heath, Jr., V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," in *IEEE Jour. Select. Areas in Commun.*, vol. 26, Oct. 2008, pp. 1341–1365.

[3] A. Naru, M. Lopez, M. Trott, and G. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," in *IEEE Jour. Select. Areas in Commun.*, vol. 16, no. 8, 1998, pp. 1423–1436.

[4] J. Roh and B. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: a VQ-based approach," in *IEEE Trans. on Info. Theory*, vol. 52, no. 3, 2006, pp. 1101–1112.

[5] P. Xia and G. Giannakis, "Design and analysis of transmit-beamforming based limited-rate feedback," in *IEEE Trans. on Signal Processing*, vol. 54, no. 5, 2006, pp. 1853–1863.

[6] D. J. Love, R. W. Heath, Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," in *IEEE Trans. on Info. Theory*, vol. 49, no. 10, 2003, pp. 2735–2747.

[7] D. J. Love and R. W. Heath, Jr., "Limited feedback unitary precoding for spatial multiplexing systems," in *IEEE Trans. on Info. Theory*, vol. 51, no. 8, 2005, pp. 2967–2976.

[8] 3GPP, "Physical layer aspects of UTRA high speed downlink packet access," in *Technical Report TR25.814, 2006*.

[9] B. Clerckx, Y. Zhou, and S. Kim, "Practical codebook design for limited feedback spatial multiplexing," in *IEEE ICC*, May 2008, pp. 3982–3987.

[10] J. S. Dyer and B. Natarajan, "Designing DS-CDMA spreading sequences via a low-complexity deterministic approach," in *IEEE Trans. on Wireless Commun.*, vol. 6, Jan. 2007, pp. 302–312.

[11] G. L. Stuber, "Pinciples of Mobile Communication," 2nd ed. Kluwer Academic Publishers, 2001.

[12] F. Bohagen, P. Orten, and G. E. Oien, "Construction and capacity analysis of high-rank line-of-sight MIMO channels," in *IEEE WCNC*, vol. 1, Mar. 2005, pp. 432–437.

[13] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," in *European Trans. on Telecomm.*, vol. 10, no. 6, Dec. 1999, pp. 585–595.

[14] W. Dai, Y. Liu, and B. Rider, "Quantization bounds on Grassmannian manifolds and applications to MIMO communications," in *IEEE Trans. on Info. Theory*, vol. 54, Mar. 2008, pp. 1108–1123.

[15] J. Kim and D. J. Park, "Constrained codebook design for a MISO beamforming system in a Rician channel," in *IEEE VTC Spring 2009*, Apr. 2009, pp. 1–5.