Performance of Spread OFDM with LDPC Coding in Outdoor Environments

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***Abstract*— In this paper, the performance of OFDM systems that utilize coding and spreading is presented. The proposed systems operate in outdoor environments and achieve coding gains through the use of Low-Density Parity-Check (LDPC) encoding. It is shown here that by introducing Hadamard-Walsh spreading in coded OFDM, additional gain in performance can be attained relative to uncoded as well as LDPC coded OFDM systems. Comparable systems using convolutional codes are shown to have similar performance, although the OFDM systems require additional interleaving and deinterleaving.**

***Index Terms*— OFDM, spreading, fading, multipath, LDPC**

I. Introduction

OFDM has emerged as a strong enabling technology for next generation high data-rate wireless applications such as Wireless Local Area Networks (WLANs), Personal Area Networks (PANs) and Fourth Generation (4G) wireless networks. Although OFDM can restrain burst errors in fading channels as it transmits data over multiple carriers, frequency selective fading can still deteriorate the data on carriers where the fading occurs. Therefore, some form of error correction is required.

Recently, spread spectrum techniques that better exploit diversity have been merged with the OFDM architecture [1][2] via an approach called spread OFDM (SOFDM) [3][4]. By spreading the data across all subcarriers, a performance gain is achieved over OFDM. In this paper, we attempt to better exploit both frequency diversity gains as well as coding gains by introducing Low-Density Parity-Check (LDPC) codes [5] in spread OFDM. Specifically, we analyze the bit-error-rate performance of the coded (LDPC and convolutional) spread OFDM systems in outdoor environments. The performance gains relative to uncoded as well as coded OFDM systems are provided

Recent advances in error-correcting codes have shown that using iterative decoders achieves performances approaching Shannon's limit. First with the introduction of Turbo Codes [6] and later with the rediscovery of LDPC codes [7], substantial coding gains can now be obtained in systems incorporating these codes into their structure.

To achieve system performance approaching Shannon's capacity, both Turbo and LDPC codes require large block sizes ($\simeq$ 10,000–40,000 bits) [6] [7]. This is not practical in wireless systems where long delays in the decoding process would be unacceptable. Therefore, it is necessary for wireless systems to utilize good codes with smaller block lengths.

This paper analyzes the performance of coded spread OFDM systems in outdoor environments. Different configurations have been used to better demonstrate the effects of coding and diversity gain on system performance. Each system uses BPSK subcarrier modulation with LDPC block sizes of 1024 bits. Systems using coding as well as spreading in their architecture outperform the others.

The rest of the paper is organized as follows: In section II we give an overview of the system, including the transmitter, the channel and the receiver. Section III presents the results and section IV gives the concluding remarks.

II. System

The LDPC coded spread OFDM system model is shown in Figure 1. Here, the incoming binary data stream first enters an LDPC encoder. LDPC codes are linear block codes defined by a very sparse parity-check matrix $\mathbf{H}$ (typically over GF(2)). A code with rate K/N is defined by the number of input bits K in a block and the number of output bits N. Matrix $\mathbf{H}$ is required to be full rank, with dimensions $M \times N$, where $M = N - K$. Regular LDPC codes are defined by a constant row weight of w_r and a column weight of w_c, where $w_c \ll M$ and $w_r = w_c N/M$ [7]. Therefore, $\mathbf{H}$ has a small number (density) of ones, giving the code its name. If the number of ones per column or row is not constant, then the code is an irregular code [8]. In this paper, we consider regular LDPC codes. In order to avoid low-weight codewords we ensure that no two columns in the $\mathbf{H}$ matrix overlap in more than one non-zero bit position.

In typical OFDM systems, the output of the LDPC encoder is modulated (using PSK or QAM) and transmited over N carriers in parallel. However, in a spread OFDM system, the modulated data symbols are first spread using Hadamard-Walsh (HW) codes. The spread data symbols are then transmitted over N carriers (implemented using an IFFT block followed by a Digital-to-Analog (D/A) converter). As a result of this spreading, each data symbol resides on all carriers enabling the system to better exploit the available frequency diversity. Therefore, the transmitted signal corresponds to

$$s(t) = \Re e\left\{ \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} b_k c_k^i e^{j(2\pi f_c t + 2\pi i \Delta f t)} \right\}, \tag{1}$$

where b_k is the data symbol (assuming BPSK modulation $b_k = \{\pm 1\}$), c_k^i is the ith chip of the kth spreading code

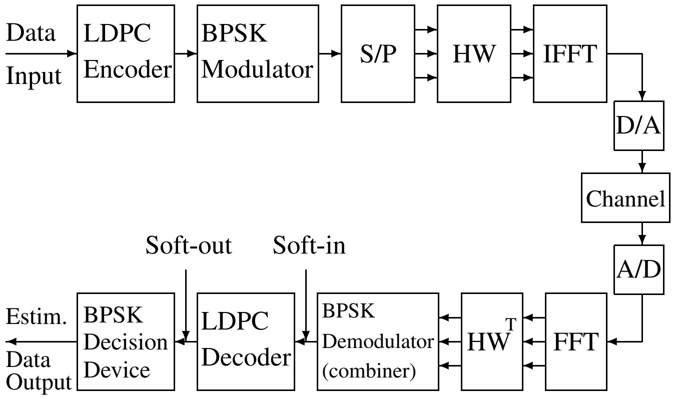

Fig. 1. LDPC coded spread OFDM system.

in the HW code set scaled to ensure a total energy per bit equal to 1 (i.e. $c_k^i = \{\pm\frac{1}{\sqrt{N}}\}$), Δf is the subcarrier spacing ensuring orthogonality among subcarriers ($\Delta f = \frac{1}{T_s}$, where T_s is the symbol duration), and f_c is the carrier frequency. Practical implementation of OFDM and spread OFDM may permit the use of a cyclic prefix in order to avoid Inter-Symbol Interference (ISI) due to multipath.

The transmitted signal in equation (1) is sent across a slowly varying multipath channel. Multipath propagation in time results in frequency selectivity over the entire bandwidth of transmission [9]. However, each narrowband subcarrier in OFDM experiences a non-selective fade, with channel fades being correlated among different subcarriers. The correlation between the ith subcarrier fade and the jth subcarrier fade is given by [10]

$$\psi_{i,j} = \frac{1}{1 + ((f_i - f_j)/(\Delta f)_c)^2}, \tag{2}$$

where $(\Delta f)_c$ is the coherence bandwidth of the channel. The correlated Rayleigh fades are generated according to the methods discussed in [11].

The received signal can be modeled as

$$r(t) = \Re e\left\{ \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} \alpha_i b_k c_k^i e^{j(2\pi f_c t + 2\pi i \Delta f t + \phi_i)} \right\} + n(t). \tag{3}$$

Here, α_i is the Rayleigh fading gain, ϕ_i is the phase offset introduced in the ith subcarrier due to the channel and $n(t)$ is the additive white gaussian noise (AWGN).

Signal $r(t)$ is first projected onto the N orthogonal subcarriers to yield the vector $\underline{r} = (r_0, \cdots, r_{N-1})^T$. This is practically implemented using an N-point FFT. The ith component of $\underline{r}$ corresponds to

$$r_i = \alpha_i \sum_{k=0}^{N-1} b_k c_k^i + n_i \qquad i = 0, \cdots, N-1, \tag{4}$$

where n_i is the noise per carrier with variance σ_n^2. We have assumed perfect phase tracking and removal while writing equation (4). In order to detect the jth symbol, $\underline{r}$ is despread using the jth HW spreading code. This results in $\underline{r}(j) = (r_0(j), \cdots, r_{N-1}(j))^T$, where $r_i(j)$ is equal to

$$r_i(j) = \frac{\alpha_i b_j}{N} + \sum_{k \neq j} \alpha_i b_k \rho_{kj}^i + c_j^i n_i \quad i, j = 0, \cdots, N-1. \tag{5}$$

Here, the first term is the desired information and the second term represents the ISI on the ith carrier ($\rho_{kj}^i = c_k^i c_j^i$).

Next, a suitable combining strategy is used to linearly combine the $r_i(j)$ to arrive at the decision variable $R(j)$. That is

$$R(j) = \sum_{i=0}^{N-1} \omega_i r_i(j). \tag{6}$$

In this work, we employ MMSEC as it has been proven to produce the best performance in terms of probability of error [12]. The MMSEC method approximates the data symbol b_k from the received vector $\underline{r}(j)$ using equation (6). Based on the MMSE criterion, the estimation error must be orthogonal to $r_i(j)$ [13]. That is,

$$E\Big\{\Big(b_k - \sum_{i=0}^{N-1} \omega_i r_i(j)\Big) \cdot r_i(j)\Big\} = 0 \qquad i = 0, \cdots, N-1, \tag{7}$$

where $E\{\cdot\}$ denotes the expected value. The solution to equation (7) as obtained from Weiner filter theory corresponds to [14]

$$\omega_i = C^{-1} A, \tag{8}$$

where

$$C = E\{r_i(j) \cdot r_i(j) | \alpha_i\} \tag{9}$$

and

$$A = E\{b_k \cdot r_i(j) | \alpha_i\}. \tag{10}$$

This operation, when applied to equation (5) yields

$$C^{-1} = \frac{1}{\sigma_{r_i(j)}^2} \tag{11}$$

and

$$A = \frac{\alpha_i}{N}, \tag{12}$$

where

$$\sigma_{r_i(j)}^2 = \frac{\alpha_i^2 + \sigma_n^2}{N}. \tag{13}$$

Therefore, the optimal weight vector $\underline{\omega} = (\omega_0, \cdots, \omega_{N-1})$, which is identical for all data symbols, corresponds to

$$\omega_i = \frac{\alpha_i}{\alpha_i^2 + \sigma_n^2}. \tag{14}$$

In practice, the despreading and linear combining operations can be combined into a single matrix transformation resulting in reduced complexity. This linear transformation, $\mathbf{M}$, is given by

$$\mathbf{M} = \begin{pmatrix} c_1^1\omega_1 & c_2^1\omega_2 & \cdots & c_N^1\omega_N \\ c_1^2\omega_1 & c_2^2\omega_2 & \cdots & c_N^2\omega_N \\ \vdots & \vdots & \ddots & \vdots \\ c_1^N\omega_1 & c_2^N\omega_2 & \cdots & c_N^N\omega_N \end{pmatrix} \tag{15}$$

The N soft decision statistics corresponding to the N data symbols, $\underline{R} = (R(0), \cdots, R(N-1))^T$ are obtained from $\underline{r}$ via the transformation

$$\underline{R} = \mathbf{M}\underline{r}. \tag{16}$$

This $\underline{R}$ vector is then fed into the LDPC decoder.

The LDPC decoder uses the sum-product algorithm, which is an iterative message passing algorithm. Decoding is performed until a valid decoding output is obtained or a maximum number of iterations set is reached. In this work, we employ an LDPC decoder similar to that in [7]. The log-likelihood ratio of the data symbols is also updated, taking into account the characteristics of the channel. The derivation of the log-likelihood ratio is detailed below:

The decision statistic for the jth data symbol is given as

$$\begin{aligned} R(j) &= \frac{b_j}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i + \sum_{i=0}^{N-1}\omega_i\alpha_i\sum_{k\neq j} b_k\rho_{kj}^i \\ &+ \sum_{i=0}^{N-1} c_j^i n_i\omega_i \end{aligned} \tag{17}$$

$$= \frac{b_j}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i + \mu_j. \tag{18}$$

Assuming that N is large (which is true in most OFDM systems), the interference term (second term in equation (17)) can be approximated as a Gaussian random variable (by invoking the central limit theorem). Therefore, μ_j is also Gaussian with probability density function (PDF)

$$p(\mu_j) = \frac{1}{\sqrt{2\pi\sigma_{\mu_j}^2}}\exp\left[\frac{-\left(R(j) - \frac{b_j}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i\right)^2}{2\sigma_{\mu_j}^2}\right], \tag{19}$$

where $\sigma_{\mu_j}^2$ is the variance of μ_j and is equal to

$$\sigma_{\mu_j}^2 = \frac{1}{N^2}\sum_{k\neq j}\Big(\sum_{i=0}^{N-1}\alpha_i\omega_i\rho_{kj}^i\Big)^2 + \frac{\sigma_n^2}{N}\sum_{i=0}^{N-1}\omega_i^2. \tag{20}$$

The log-likelihood ratio (LLR) for the data symbol, b_j, given $R(j)$ and the channel corresponds to

$$LLR_{SOFDM}(j) = \log\frac{P(b_j = 1|R(j),\alpha_i)}{P(b_j = -1|R(j),\alpha_i)}. \tag{21}$$

Substituting equation (19) into equation (21) gives

$$LLR_{SOFDM}(j) = \log\left\{\frac{\exp\left[\frac{-\left(R(j)-\frac{1}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i\right)^2}{2\sigma_{\mu_j}^2}\right]}{\exp\left[\frac{-\left(R(j)+\frac{1}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i\right)^2}{2\sigma_{\mu_j}^2}\right]}\right\}, \tag{22}$$

which simplifies to

$$LLR_{SOFDM}(j) = \frac{2\frac{R(j)}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i}{\sigma_{\mu_j}^2}. \tag{23}$$

Therefore, the LLR can be written as

$$LLR_{SOFDM}(j) = \frac{\frac{2R(j)}{N}\sum_{i=0}^{N-1}\alpha_i\omega_i}{\frac{1}{N^2}\sum_{k\neq j}\left(\sum_{i=0}^{N-1}\alpha_i\omega_i\rho_{kj}^i\right)^2 + \frac{\sigma_n^2}{N}\sum_{i=0}^{N-1}\omega_i^2}. \tag{24}$$

In this work, we assume perfect channel information to calculate the $LLR_{SOFDM}(j)$.

III. Results

Figures 2-4 show the performance results of the OFDM system with and without coding and spreading. Rate 1/2 regular LDPC and constraint length 7 convolutional codes are used; the number of carriers, N, is set to 1024; and BPSK modulation is considered. In addition, OFDM systems employing convolutional codes are interleaved and deinterleaved at the transmitter and receiver ends, respectively. It is also assumed that the channel is perfectly known to the receiver. The system has a total bandwidth, BW, of 5 MHz., and uses the Hilly Terrain (HT), Typical Urban (TU) and Rural Area (RA) outdoor channel models [15]. The channel parameters are given in Table I.

TABLE I
Channel parameters

Channel	$(\Delta f)_c$
HT	39.72 kHz.
TU	188 kHz.
RA	2050 kHz.

The figures demonstrate that the performance of uncoded OFDM improves with the use of spreading, with the largest improvement seen in the Hilly Terrain channel. This is because the HT channel has the least coherence bandwidth and therefore the maximum frequency diversity to exploit, among the three channels considered. By employing LDPC encoding the performance of spread OFDM can further be enhanced. Specifically, the LDPC coded spread OFDM systems on all three channels offer about 1 dB gain relative to LDPC coded OFDM at probability of errors less than 10^{-3}.

The Typical Urban and Rural Area OFDM and spread OFDM systems using convolutional codes perform as well as similar LDPC coded systems. On the other hand, in the Hilly Terrain channel LDPC coded systems are superior, resulting in up to 1.5 dB gain at 10^{-5} bit-error-rate.

Without interleaving, the performance of OFDM systems using convolutional codes is worse than LDPC coded systems. This is a disadvantage that would create additional complexity in the architecture for systems employing convolutional encoding. It is also important to note that for systems where decoding delay is less critical, increasing the LDPC code block length will further enhance the performance.

IV. Conclusions

In this paper, the performance of a wireless system employing LDPC codes and spread OFDM in various outdoor channels has been examined. This system performed very well for both the Hilly Terrain and Typical Urban channel models, but not as well for the Rural Area channel. For all channels, the use of spreading improved the performance signficantly compared to OFDM systems without spreading. Adding LDPC block encoding provided even greater performance gains. Therefore, a good spreading method and a code with excellent error-correcting capabilities would provide significant gains over uncoded systems. LDPC codes are shown to be good candidates for systems that strive for error-coding capabilities with less complex architecture that can also perform well with relatively small block sizes. Convolutional codes are also shown to offer similar performance gains, but the gain of LDPC codes will improve as the block size is allowed to increase.

Future work on spread OFDM will involve channel estimation algorithms and/or Bayesian approaches to evaluate the log-likelihood ratio for spread OFDM. In the area of LDPC coding, additional work will compare different spreading strategies along with the effect of using irregular LDPC codes.

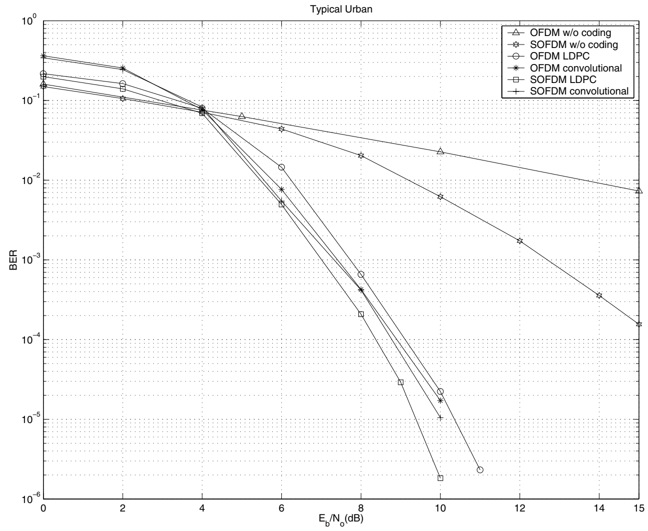

Fig. 3. Performances of uncoded and LDPC coded OFDM and spread OFDM systems in Typical Urban channel.

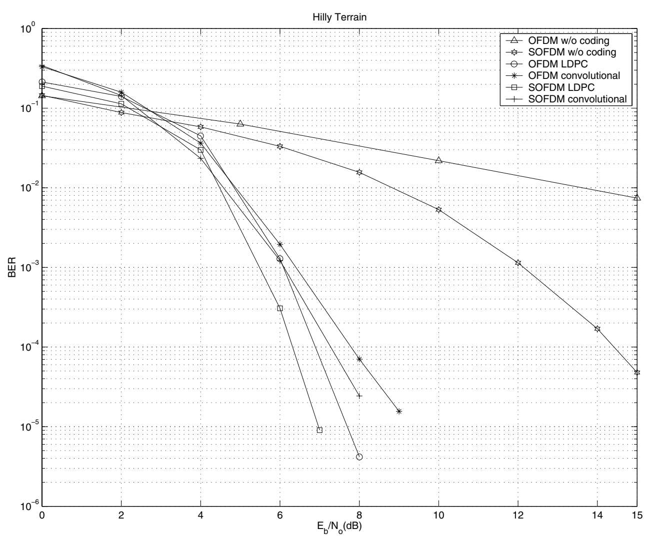

Fig. 2. Performances of uncoded and LDPC coded OFDM and spread OFDM systems in Hilly Terrain channel.

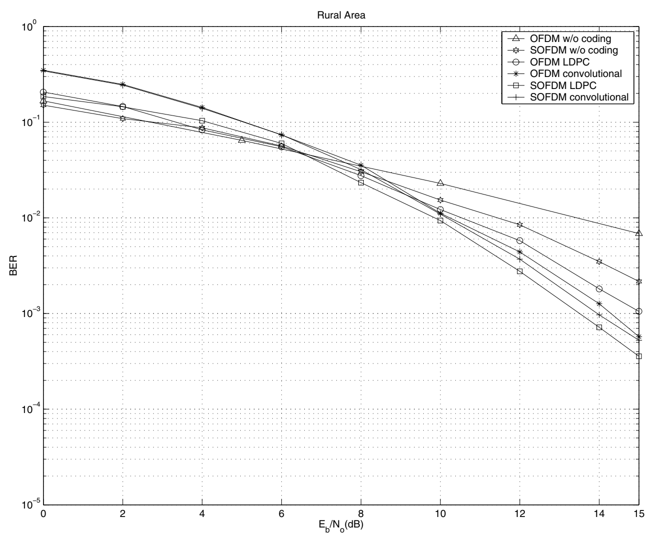

Fig. 4. Performances of uncoded and LDPC coded OFDM and spread OFDM systems in Rural Area channel.

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