Residue Number System Arithmetic-Inspired Hopping-Pilot Pattern Design

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***Abstract*—In this paper, we propose a novel application of residue number system (RNS) arithmetic in designing hopping-pilot patterns for cellular downlink orthogonal frequency-division multiple access (OFDMA). By hopping the scan lines in either the time or the frequency domain, RNS-based pilot patterns fulfill the Nyquist sampling theorem. That is, by using RNS-based pilot patterns, the channel's delay–Doppler response can fully be reconstructed without severe aliasing. In addition to channel estimation, our proposed scheme can achieve other objectives, such as device/cell identification and time–frequency synchronization. We show that the RNS-based approach has more pairs of hopping–pilot patterns that are collision free than the Costas-array-based method. This is helpful in not only mitigating intra-/intercell interference but also identifying multiple devices in a multicell multiantenna environment. Moreover, hopping in time increases the pilot's time support, which, in turn, enables quick initial acquisition of time–frequency offsets.**

***Index Terms*—Downlink orthogonal frequency-division multiple access (OFDMA), hopping-pilot pattern, intra-cell interference, residue number system arithmetic.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has widely been accepted as an enabling technology for next-generation wireless communication systems [1]. In OFDM, high-rate data streams are broken down into a number of parallel lower rate streams that are transmitted on orthogonal subcarriers (frequencies). OFDM also forms the foundation for a multiple-access (MA) scheme, termed orthogonal frequency-division MA (OFDMA), which is being considered for evolved third-generation cellular systems E-UTRA [2]. To acquire channel-state information at the receiver to facilitate coherent detection, pilot-assisted channel-estimation techniques are generally used in OFDM/OFDMA [3]. In a pilot-aided transmission scheme, a limited number of known signals referred to as pilots are placed across time and frequency domains. Essentially, such 2-D pilot signals sample the channel's time–frequency response in a way that the full channel's response can be reconstructed without severe aliasing. Moreover, in cellular OFDMA systems, different pilot patterns have to be allocated to different base stations while minimizing the number of collisions among the patterns. This is because, if all transmitted pilot patterns across multiple cells are the same, they interfere with each other, resulting in degraded system throughput and poor channel estimation quality. Hence, there is a need for a large number of distinct pilot patterns, with each pair of them having a minimal number of collisions. Additionally, it is important to capture the entire channel response with a limited number of 2-D pilot signals. In the context of OFDM, to fulfill the Nyquist sampling theorem, there exist both a minimum subcarrier spacing and a minimum symbol spacing between pilots.

Manuscript received December 29, 2009; revised March 11, 2010 and May 22, 2010; accepted May 22, 2010. Date of publication June 7, 2010; date of current version September 17, 2010. The review of this paper was coordinated by Dr. K. T. Wong.

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Digital Object Identifier 10.1109/TVT.2010.2051692

In [4], Costas-array-based pilot patterns are investigated. Guey [4] showed that Costas-array-based 2-D pilot patterns have a minimal number of collisions with their arbitrary time–frequency-shifted versions. In his later work [5], both link- and system-level performances of Costas-array-based pilot patterns are evaluated. However, the number of unique pilot patterns that are generated via the Costas array is still limited, particularly when the system is not synchronized in time. A Latin square-based pilot pattern design is proposed and investigated in [6] and [7]. However, wide variations in the system-level performance are observed in the Latin square-based scheme, which highly limits its application.

In this paper, we invoke the use of residue number system (RNS) arithmetic to determine 2-D pilot patterns with limited mutual interference. The RNS has received broad attention in both computer computations and communications in recent years [8]–[10]. However, the use of the RNS in constructing a large set of multiple 2-D pilot patterns with low cross correlation has not been investigated in any of the prior efforts. Therefore, understanding and quantifying the role of the RNS in pilot pattern design are the objectives of this effort. We show that, by hopping the scan lines in either the time or the frequency domain, RNS-based pilot patterns fulfill the Nyquist sampling theorem. Furthermore, in contrast to the Costas array, the RNS-based method has more pairs of pilot patterns that have zero collision. Additionally, RNS-based pilot patterns allow quick initial time–frequency synchronization due to the fact that hopping in time increases the time support in each pilot period. This, in turn, enables acquisition of time–frequency offsets within a very short observation.

The rest of this paper is organized as follows: RNS arithmetic is introduced in Section II. In Section III, detailed procedures of constructing RNS-based pilot patterns are presented, incorporating analysis of their desired properties. Illustrative examples are shown in Section IV. Finally, we conclude our paper in Section V.

II. Residue Number System Arithmetic

RNS arithmetic is defined by the choice of v number of positive integers m_i $(i = 1, 2, \ldots, v)$, which is referred to as moduli [8]. If all the moduli are pairwise relative primes to each other, any integer N_k that falls in the range of $[0, M)$ can uniquely and unambiguously be represented by the residue sequence $(r_{k,1}, r_{k,2}, \ldots, r_{k,v})$, where $M = \prod_{i=1}^{v} m_i$. $[0, M)$ is defined as the dynamic range of the RNS. Mathematically, the preceding descriptions can be expressed as

$$N_k \Longleftrightarrow (r_{k,1}, r_{k,2}, \ldots, r_{k,v}) \quad (1)$$

$$r_{k,i} = N_k \bmod \{m_i\}, \qquad i = 1, 2, \ldots, v. \quad (2)$$

III. Residue Number System-Based Hopping Pilot Pattern Design

From the standpoint of sampling theory, to fully reconstruct the channel's delay–Doppler response, its time–frequency transform has to be sampled (at least) at Nyquist rates in both time and frequency domains. We can also interpret this in a broader sense. That is, in the time domain, the sampling frequency $1/T_p$ should not be less than the channel's maximal Doppler spread $\nu_{\max}$, where T_p represents the spacing between two consecutive pilot probes in time. Similarly, in the frequency domain, the sampling frequency $1/f_p$ must not be less than the channel's maximal delay spread $\tau_{\max}$, where f_p denotes the spacing between two consecutive pilot signals in frequency. In the context of OFDM, typical comb-type pilot patterns are widely used, where pilot signals sample the channel's time–frequency response at Nyquist rates [11]. However, a typical comb-type pilot pattern design lacks generality, and this limits its application in identifying devices/cells, particularly when the system is not time synchronized. In this paper, we invoke the use of the RNS to design 2-D pilot patterns with limited mutual interference. The resulting pilot signals not only sample the channel's time–frequency response at exact Nyquist rates but generate a much larger set of unique pilot patterns relative to regular comb-type and Costas-array-based pilot patterns as well.

Assume that G represents the required number of OFDM symbols between two consecutive pilot signals in time and that M corresponds to the number of OFDM subcarriers between two consecutive pilots in frequency (both following the Nyquist sampling rate criteria). Therefore, if T_s is one OFDM symbol duration and f is one subcarrier spacing, we have $T_p = GT_s$ and $f_p = Nf$. Based on these assumptions, detailed design procedures of our proposed RNS-based pilot patterns are given here.

1) Partition the total available subcarriers N into M_c clusters, with each cluster containing M number of contiguous subcarriers (i.e., $N = M \cdot M_c$).
2) If M can be written as a product of two pairwise relative primes, e.g., $M = a \times b$, then within each cluster, we can group a subclusters with b subcarriers in each subcluster.
3) Index the subcarriers in each subcluster from 0 to $b - 1$.
4) Index the subclusters in each cluster from 0 to $a - 1$.
5) At the zeroth time slot, assign integer N_k as the initial address (IA) of pilot signals, where $0 \leq N_k < M$.
6) If $N_k \bmod \{a, b\} = \{\hat{a}, \hat{b}\}$, then the $\hat{b}$th subcarrier out of the $\hat{a}$th subcluster is selected for transmitting the pilot signal within one cluster.
7) Performing step 6) through all M_c clusters, M_c pilot signals are obtained with M OFDM subcarriers between each pair of the pilot signals.
8) At the t_sth time slot, assign integer $N_k + t_s \bmod \{G\}$ as the current address (CA) of pilot signals and repeat steps 6) and 7).
9) Repeat steps 6)–8) until one mutually orthogonal pilot pattern is obtained.
10) If M can be expressed as products of w different combinations of two pairwise relative primes, then w unique pilot patterns can be obtained by repeating steps 2)–9) w times.

One illustrative example of RNS-based pilot pattern design is given in Fig. 1. In this example, we assume that $N = 12$, $M_c = 2$, $M = 6$, and $G = 4$. Straightforwardly, $M = 6 = 2 \times 3$. Therefore, if the IA of pilot signals is 4 (at the zeroth time slot), we have $4 \bmod \{2, 3\} = \{0, 1\}$. This indicates that the first subcarrier out of the zeroth subcluster within each single cluster is extracted out for transmitting pilot signals. This corresponds to the actual first and seventh subcarriers in the zeroth OFDM symbol. At the first time slot, the CA of pilot signals is calculated as $4 + 1 \bmod \{4\} = 5$. Straightforwardly, $5 \bmod \{2, 3\} = \{1, 2\}$. Hence, the second subcarrier out of the first subcluster within each single cluster is selected for pilot signals. This actually corresponds to the fifth and eleventh subcarriers in the first OFDM symbol. Fig. 2 shows the corresponding pilot pattern that is generated using the previously described RNS arithmetic. The IAs (i.e., four in this example) of pilot signals are marked at the selected positions for simplicity. In general, the entire RNS-based pilot pattern is obtained by extending the generic $M \times G$ pilot pattern in both time (see the dashed bidirectional arrow) and frequency (see the solid bidirectional arrow). We note that the algorithm previously presented assumes that M can be written as a product of *two* pairwise relative primes. However, the algorithm is also suited for the case when M can be written as a product of *multiple* (larger than two) pairwise relative primes. Under this scenario, more orthogonal pilot patterns

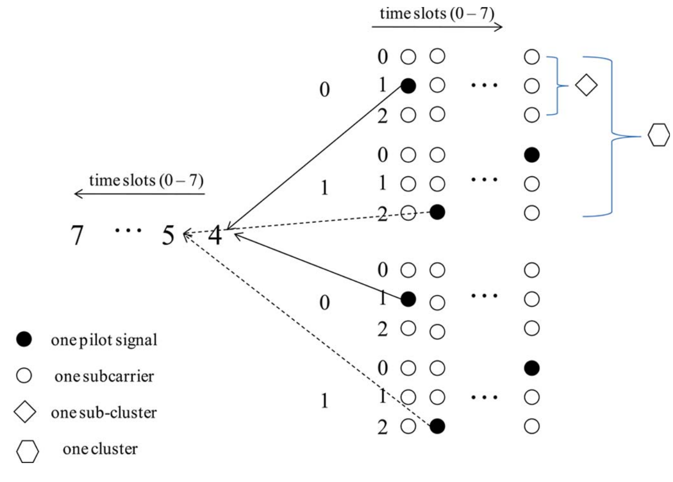


Fig. 1. Design procedures of pilot patterns using RNS arithmetic.

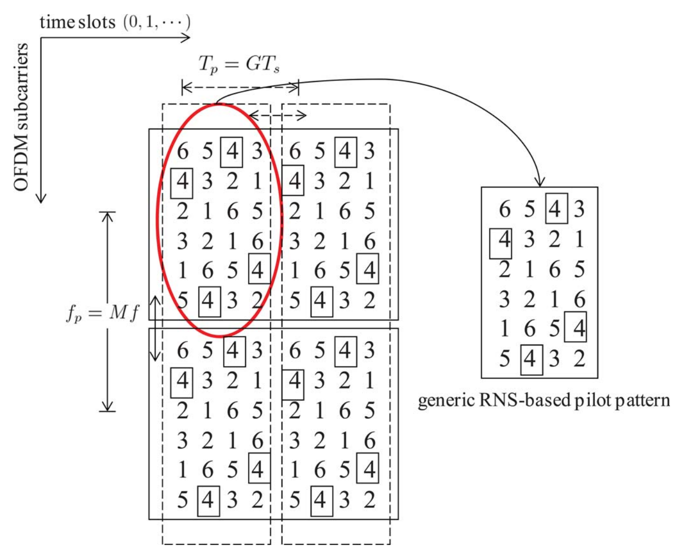


Fig. 2. Proposed RNS-based pilot pattern.

can be generated. The total number of orthogonal patterns that can be obtained via the RNS arithmetic can be found in [10]. It is worth noting here that our proposed approach is particularly suited for the case that M can be written as a product of at least two primes. If M cannot be divided into at least two primes, a regular cyclic shift can be applied to obtain the generic hopping pilot patterns.

Some important features of our proposed RNS-based pilot patterns are presented next.

Lemma 1: The pilot patterns designed using the RNS arithmetic are orthogonal.

Proof: Orthogonality means that pilot patterns that are obtained using different IAs will not collide with each other. To prove that RNS-based pilot patterns are orthogonal, we need to show that every N_k in the range of $[0, M)$ has a unique residue set that is different from residue sets generated by other integers within the same range. We will prove this by contradiction as follows.

Assume that N_1 and N_2 are different integers that are in the same range of $[0, M)$ with the same residue set. That is

$$N_1 \bmod \{m_i\} = N_2 \bmod \{m_i\}, \qquad i = 1, 2, \ldots, v. \tag{3}$$

Therefore, we have

$$(N_1 - N_2) \bmod \{m_i\} = 0. \tag{4}$$

Thus, we can conclude from (4) that $N_1 - N_2$ is actually the least common multiple (LCM) of m_i. Furthermore, if m_i are pairwise relative primes to each other, their LCM is $M = \prod_{i=1}^{v} m_i$, and it must be that $N_1 - N_2$ is a multiple of M. However, this statement does not hold since $N_1 < M$ and $N_2 < M$. Therefore, by contradiction, N_1 and N_2 should not have the same residue set. ∎

Lemma 2: The maximal number of collisions between any two RNS-based pilot patterns under arbitrary mutual nonzero periodic time shift is one.

Proof: Exploiting the maximal number of collisions between any two *generic* RNS-based pilot patterns under arbitrary time shift would suffice as the entire RNS-based pilot patterns are simply time–frequency expansions of generic patterns. In the context of OFDM, the generic pilot pattern can be represented by an integer sequence that contains indexes of corresponding OFDM subcarriers across time, which is termed the RNS sequence. For instance, in Fig. 1, the generic RNS sequence is {1, 5, 0, 4}. Therefore, to characterize the maximal number of collisions between each pair of the RNS sequences, we first look into the general solution that is derived based on finite field theory, which includes RNS sequences as a special case.

An integer sequence can be defined as a set of G frequencies from a finite set of Q frequencies. If Q is prime, a one-to-one representation between the set of Q frequencies and the Galois field of Q elements (i.e., GF(Q)) can be obtained. In other words, the integer sequence of length G $(f_i, i = 0, 1, \ldots, G-1)$ can be generated by the corresponding associated polynomials of at most degree d, which is a set of elements in GF(Q) [7]. Equivalently, the integer sequence can be expressed as

$$f_i = P(i), \qquad i = 0, 1, \ldots, G-1 \tag{5}$$

$$P(x) = \sum_{j=0}^{d} n(j) x^j \tag{6}$$

where Q is prime, coefficient $n(d-1)$ is fixed for all associated polynomials, and all other $n(j)$ can be chosen as all possible values in GF(Q) [12].

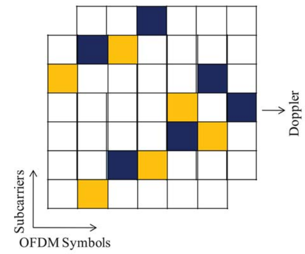
In general, the number of collisions between two different sequences is defined as the cross correlation of these two sequences. Assume p and r as two different integer sequences and that their associated polynomials are $P(x)$ and $R(x)$, respectively. Then, the difference between these two polynomials can be calculated as

$$E(x) = P(x) - R(x) = \sum_{j=0}^{d} e(j) x^j. \tag{7}$$

From (7), we can see that $E(x)$ is at most of degree d. This indicates that sequences p and r are identical to each other at most d positions (therefore resulting in at most d roots of $E(x)$). From the cross correlation's point of view, (7) implies that the cross correlation of two different integer sequences is no more than d, which is equivalent to saying that the maximal number of collisions between two sequences is no more than d. For $d = 1$, the set of linear congruence sequences is obtained, which includes RNS sequences as a special case [8]. Therefore, the number of collisions between any two RNS-based pilot patterns under arbitrary nonzero time shift is no more than $d = 1$. ∎

Lemma 3: If the system is perfectly time synchronized, the number of RNS-based pilot patterns that have one collision with their arbitrary time-shifted versions is wMG^2. For the case when the system is not synchronized in time, this number becomes wMG.

Proof: The number of unique RNS-based pilot patterns obtained via one $M \times G$ generic pilot pattern is MG. Moreover, if M can be written as products of w different combinations of pairwise relative primes, wMG unique pilot patterns are obtained, with each of them having a maximal number of collisions as one. Under perfect time synchronization, G time-shifted versions can also be taken into account. Under this scenario, the number of unique RNS-based pilot patterns is wMG^2. ∎



											Doppler
0	0	1	1	0	0	1	0	0	0	0	5
0	0	0	0	1	0	0	0	0	0	0	4
0	0	2	0	0	0	0	0	1	0	0	3
0	0	0	1	0	0	1	0	0	1	0	2
0	2	0	0	1	0	0	3	0	0	0	1
0	0	0	0	0	6	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	1	0	-1
0	0	0	0	3	0	0	1	0	0	1	-2
0	0	1	0	0	0	0	0	2	0	1	-3
0	0	0	1	0	0	2	0	0	0	0	-4
0	0	0	0	0	0	0	0	0	0	0	-5
-5	-4	-3	-2	-1	0	1	2	3	4	5	delay

Fig. 3. Sidelobe array distribution of the RNS-based pilot pattern of length 6.

It is important to note that, in typical comb-type and Costas-array-based pilot pattern designs, the numbers of unique pilot patterns are MG and MG^2, respectively (assuming perfect time synchronization). Lemma 3 shows that our proposed RNS-based method significantly enhances the degree of freedom for generating unique pilot patterns under arbitrary nonzero time shift, particularly when w is large. This is extremely helpful in terms of two aspects: 1) interference mitigation and 2) cell/device identification in a cellular OFDMA system. With regard to the first aspect, our proposed method can support a large number of cells, with each of them employing a unique RNS-based pilot pattern. This way, the number of intercell pilot-to-pilot collisions can be minimized; however, in the Costas-array-assisted scheme, to support the same number of cells, the same pilot patterns have to be reused due to the lack of generality. For the second aspect, cells with different hopping sequences can easily be identified at the receiver. Since the RNS-based approach can generate more unique hopping-pilot sequences than other existing schemes, it is well suited for this scenario.

Lemma 1 guarantees orthogonality within a single cell. Lemma 2 reveals that, if only time shift is taken into account, our proposed scheme is ideal as the maximal number of coincidences between RNS-based pilot patterns and their arbitrary time-shifted versions is one. Specifically, if multiple antennas are employed in each of the cells, orthogonal subsets of RNS-based pilot patterns can be allocated to different transmit antennas within the same cell. This will force intracell interference to zero. Alternately, RNS-based pilot patterns that have single coincidence with each other can be allocated to adjacent cells. This way, it is ensured that intercell interference can be averaged out.

From the perspective of device/cell identification, only time-shift-based correlation property may not ensure that there are enough number of unique pilot patterns that can be assigned to multiple devices in a multicell environment. Hence, we consider shifting the RNS-based pilot patterns in both time and frequency and explore the possible number of "good" pilot patterns that can be used for device identification. In this paper, we use the ambiguity sidelobe distribution array (SDA) [4] to characterize this aspect. The SDA of a generic RNS-based pilot pattern of length 6 is given in Fig. 3. The number indicated in the nth row of the mth column of Fig. 2 corresponds to the number of collisions between the base signal and its time–frequency-shifted version, where the time shift $\tau = \pm mT_s$, and the corresponding frequency shift $\nu = \pm nf_p$ (where + and − simply imply right shift and left shift relative to the base signal, respectively).

Lemma 4: The total number of collisions in the SDA of any time hopping-pilot pattern of length L is L^2.

Proof: The SDA of a generic time hopping-pilot pattern is symmetric with respect to the origin $(m = 0, n = 0)$. Hence, calculating the total number of coincidences in the right half (or the left half) of the SDA (i.e., $m \geq 0$) will suffice. Denote the coordinates of the pilots in a time hopping-pilot pattern of length L by $\{\tau_{i,q}\}$, where $i = 1, 2, \ldots, L$, $q \in \{1, 2, \ldots, L\}$ depends on the actual pattern. Therefore, the coordinates of the pilots in the corresponding time–frequency-shifted pilot pattern (e.g., time shift m and frequency shift n) would be $\{\tau_{i+m,q+n}\}$. Straightforwardly, we have

$$Pr(\tau_{i,q} = \tau_{j+m,p+n} | j \leq i) = 1, \quad i, j = 1, 2, \ldots, L \; p, q \in \{1, 2, \ldots, L\}. \tag{8}$$

The total number of coincidences in the right half of the SDA is calculated as

$$\tilde{I} = \sum_{j=1}^{L} \sum_{i=1}^{L} Pr(\tau_{i,q} = \tau_{j+m,p+n} | j \leq i). \tag{9}$$

We can simplify (9) to get

$$\tilde{I} = \sum_{l=1}^{L-1} l + L. \tag{10}$$

Taking the left half of the SDA into account, the total number of collisions in the SDA of a time hopping-pilot pattern of length L is calculated as

$$I = 2 \sum_{l=1}^{L-1} l + L = L^2. \tag{11}$$

■

Lemma 4 reveals that, for any time hopping-pilot pattern design (including the Costas array and the RNS as special cases), the total number of collisions between the base pattern and its arbitrary time–frequency-shifted versions is fixed. Guey [4] demonstrates that the Costas-array-based pilot pattern is ideal as any pair of patterns with distinct time–frequency shifts have at most one coincidence. In contrast to the Costas array, the maximal number of pilot-to-pilot collisions between the RNS-based pilot pattern and its arbitrary time–frequency-shifted versions is larger than one (according to Lemma 4). However, the RNS-based scheme results in more pairs of pilot patterns that have zero collision (see Fig. 3). This is helpful in identifying multiple devices/cells. That is, collision-free pilot patterns can be assigned to the cells that are adjacent to the cell of interest. Patterns that have more collisions are allocated far away such that intercell interference is negligible. In this case, device/cell identification errors can significantly be reduced [13]. In this paper, only time hopping-pilot pattern design is considered. However, discussions can easily be extended to hopping in the frequency domain and is part of our future work.

IV. Simulation Results

The parameters of the simulated system are given in Table I. The cyclic prefix within one OFDM symbol duration is assumed to be long enough to eliminate intersymbol interference. A six-ray channel impulse response is considered, following the UTRA Vehicular Test Environment [14].

TABLE I
System Parameters

Transmission BW	5MHz
Carrier frequency	2GHz
OFDM symbol duration T_s	$100\mu s$
CP duration	$10\mu s$
Subcarrier spacing f	11KHz
FFT size	256
Occupied Subcarriers	240
Number of OFDM symbols per T (G)	6
Channel impulse response	6-ray UTRA
Channel coding	1/2 convolutional code
Modulation	QPSK
Channel estimator	Least Square (LS)
Number of interference cells	5
Number of transmit antenas M_t	2
Number of receive antenas M_r	1
Pilot power boost factor ρ [5]	0dB
Time Synchronization	Perfect
$p(t)$	RRC 0.5

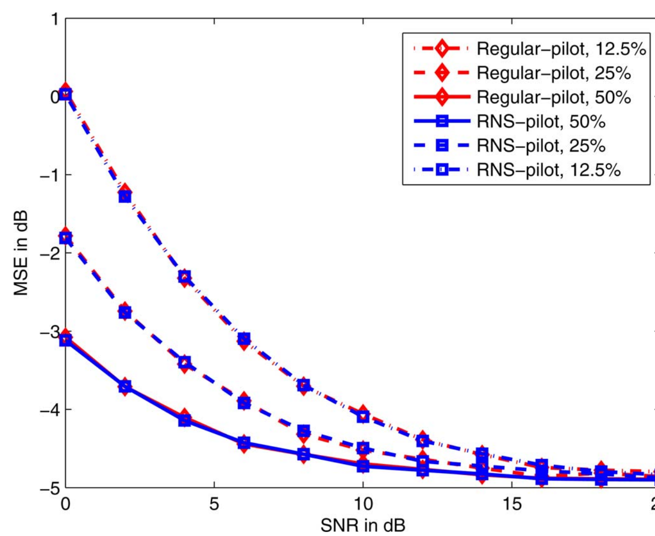


Fig. 4. MSEs of regular and RNS-based pilot-pattern-assisted channel estimation in a link-level simulation ($\rho = 0$ dB).

In Fig. 4, mean-squared error (MSE) performance of regular and RNS-based pilot pattern-aided least-squares (LS) channel estimation is evaluated. Here, MSE is defined as

$$\text{MSE} = E\left[(\hat{\mathbf{h}} - \mathbf{h})^2\right] \tag{12}$$

where $\hat{\mathbf{h}}$ is the LS estimated channel response of the actual channel response $\mathbf{h}$. Various pilot densities are taken into account during the simulation, where the pilot density is defined as the ratio between the number of pilot signals and the total number of subcarriers. It is observed that, under the same pilot density, the RNS-based pilot pattern design is identical to a regular comb-type pilot pattern in terms of MSE. This is expected as the proposed RNS-based pilot pattern satisfies the sampling requirement (i.e., Nyquist criteria). Therefore, the complete channel response can be estimated without severe aliasing. To further improve the channel-estimation performance, our proposed RNS approach can be combined with other advanced OFDM/OFDMA channel-estimation techniques (e.g., [15]) to optimize the pilots' positions while maintaining the advantages of our proposed scheme. Hence, it would be interesting and important to optimize the interface between our proposed approach and other advanced channel-estimation methods. Due to page constraint, we will not include detailed discussions in this paper.

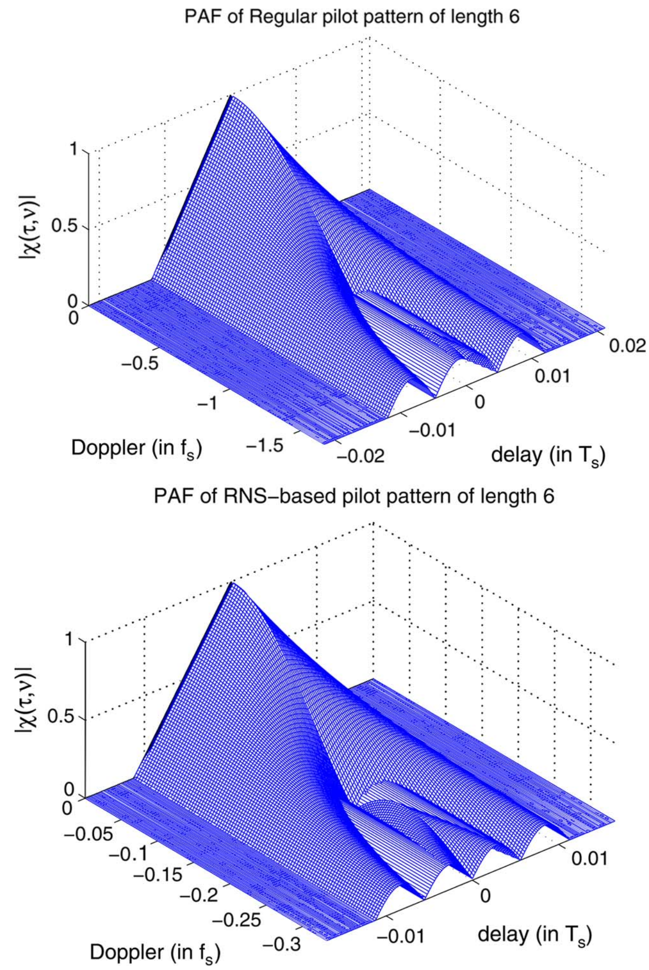


Fig. 5. PAF of regular and RNS-based pilot patterns of length 6.

RNS-based hopping-pilot pattern design enables quick time–frequency offset acquisition. Time–frequency synchronization is accomplished by correlating the output signal with the receiver's reference via a delay–Doppler correlator [4]. The delay–Doppler correlator can be written as a function of periodic ambiguity function (PAF) that is commonly used in radar detection [16]. The PAF corresponds to

$$\chi(\tau, \nu) = \int s(t) s^*(t-\tau) e^{-j2\pi\nu t} dt \tag{13}$$

where

$$s(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c(t - nT_p) e^{j2\pi m f_p t}. \tag{14}$$

Here, $c(t)$ is a base pilot signal, which is represented as

$$c(t) = \sum_{l=1}^{L} p(t - \tau_{l,q} T_s) e^{j2\pi(l-1)ft}, \qquad q \in \{1, 2, \ldots, L\} \tag{15}$$

where $p(t)$ is a narrowband pulse-shaping function given in Table I. In Fig. 5, the PAFs of the regular combi-type and RNS-based pilot pattern design of length 6 are simulated. It is observed that the RNS-based pilot pattern has a six times narrower pulsewidth of the main lobe along the Doppler axis relative to regular pilot pattern design. This enables frequency offset resolution that is six times faster than regular pilot pattern design within one pilot period.

V. Conclusion

In this paper, we have proposed a novel application of RNS arithmetic in designing hopping-pilot patterns for cellular downlink OFDMA. In contrast to the Costas-array-based approach, the RNS-based method generates more unique hopping-pilot patterns. This is helpful in not only minimizing intracell interference and averaging intercell interference but in achieving other synchronization tasks, as well, such as device/cell identification and quick time–frequency offset acquisition.

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