LOSSY CAPACITORS

1 Dielectric Loss

Capacitors are used for a wide variety of purposes and are made of many different materials in many different styles. For purposes of discussion we will consider three broad types, that is, capacitors made for ac, dc, and pulse applications. The ac case is the most general since ac capacitors will work (or at least survive) in dc and pulse applications, where the reverse may not be true.

It is important to consider the losses in ac capacitors. All dielectrics (except vacuum) have two types of losses. One is a conduction loss, representing the flow of actual charge through the dielectric. The other is a dielectric loss due to movement or rotation of the atoms or molecules in an alternating electric field. Dielectric losses in water are the reason for food and drink getting hot in a microwave oven.

One way of describing dielectric losses is to consider the permittivity as a complex number, defined as

$$\epsilon = \epsilon' - j\epsilon'' = |\epsilon|e^{-j\delta} \tag{1}$$

where

ϵ' = ac capacitivity

ϵ'' = dielectric loss factor

δ = dielectric loss angle

Capacitance is a complex number C^* in this definition, becoming the expected real number C as the losses go to zero. That is, we define

$$C^* = C - jC'' \tag{2}$$

One reason for defining a complex capacitance is that we can use the complex value in any equation derived for a real capacitance in a sinusoidal application, and get the correct phase shifts and power losses by applying the usual rules of circuit theory. This means that most of our analyses are already done, and we do not need to start over just because we now have a lossy capacitor.

Equation 1 expresses the complex permittivity in two ways, as real and imaginary or as magnitude and phase. The magnitude and phase notation is rarely used. Instead, people usually express the complex permittivity by ϵ' and $\tan\delta$, where

$$\tan\delta = \frac{\epsilon''}{\epsilon'} \tag{3}$$

where $\tan\delta$ is called either the *loss tangent* or the *dissipation factor* DF.

The real part of the permittivity is defined as

$$\epsilon' = \epsilon_r \epsilon_o \tag{4}$$

where ϵ_r is the *dielectric constant* and ϵ_o is the permittivity of free space.

Dielectric properties of several different materials are given in Table 1 [4, 5]. Some of these materials are used for capacitors, while others may be present in oscillators or other devices where dielectric losses may affect circuit performance. The dielectric constant and the dissipation factor are given at two frequencies, 60 Hz and 1 MHz. The righthand column of Table 1 gives the approximate breakdown voltage of the material in V/mil, where 1 mil = 0.001 inch. This would be for thin layers where voids and impurities in the dielectrics are not a factor. Breakdown usually destroys a capacitor, so capacitors must be designed with a substantial safety factor.

It can be seen that most materials have dielectric constants between one and ten. One exception is barium titanate with a dielectric constant greater than 1000. It also has relatively high losses which keep it from being more widely used than it is.

We see that polyethylene, polypropylene, and polystyrene all have small dissipation factors. They also have other desirable properties and are widely used for capacitors. For high power, high voltage, and high frequency applications, such as an antenna capacitor in an AM broadcast station, the ruby mica seems to be the best.

Each of the materials in Table 1 has its own advantages and disadvantages when used in a capacitor. The ideal dielectric would have a high dielectric constant, like barium titanate, a low dissipation factor, like polystyrene, a high breakdown voltage, like mylar, a low cost, like aluminum oxide, and be easily fabricated into capacitors. It would also be perfectly stable, so the capacitance would not vary with temperature or voltage. No such dielectric has been discovered so we must apply engineering judgment in each situation, and select the capacitor type that will meet all the requirements and at least cost.

Capacitors used for ac must be *unpolarized* so they can handle full voltage reversals. They also need to have a lower dissipation factor than capacitors used as dc filter capacitors, for example. One important application of ac capacitors is in tuning electronic equipment. These capacitors must have high stability with time and temperature, so the tuned frequency does not drift beyond some specified amount.

Another category of ac capacitor is the motor run or power factor correcting capacitor. These are used on motors and other devices operating at 60 Hz and at voltages up to 480 V or more. They are usually much larger than capacitors used for tuning electronic circuits, and are not sold by electronics supply houses. One has to ask for motor run capacitors at an electrical supply house like Graingers. These also work nicely as dc filter capacitors if voltages higher than allowed by conventional dc filter capacitors are required.

The term *power factor* PF may also be defined for ac capacitors. It is given by the expression

$$\text{PF} = \cos\theta \tag{5}$$

where θ is the angle between the current flowing through the capacitor and the voltage across it.

The capacitive reactance for the sinusoidal case can be defined as

$$X_C = \frac{1}{\omega C} \tag{6}$$

where $\omega = 2\pi f$ rad/sec, and f is in Hz.

In a lossless capacitor, $\epsilon'' = 0$, and the current leads the voltage by exactly 90^o. If ϵ'' is greater than zero, then the current has a component in phase with the voltage.

$$\cos\theta = \frac{\epsilon''}{\sqrt{(\epsilon'')^2 + (\epsilon')^2}} \tag{7}$$

For a good dielectric, $\epsilon' \gg \epsilon''$, so

$$\cos\theta \approx \frac{\epsilon''}{\epsilon'} = \tan\delta \tag{8}$$

Therefore, the term *power factor* is often used interchangeably with the terms *loss tangent* or *dissipation factor*, even though they are only approximately equal to each other.

We can define the apparent power flow into a parallel plate capacitor as

$$S = VI = \frac{V^2}{-jX_C} = jV^2\omega C^* = jV^2\frac{\omega A}{d}(\epsilon' - j\epsilon'') = V^2\frac{\omega A}{d}\epsilon_r\epsilon_o(j + \text{DF}) \tag{9}$$

By analogy, the apparent power flow into any arbitrary capacitor is

$$S = P + jQ = V^2\omega C(j + \text{DF}) \tag{10}$$

Table 1: Dielectric Constant ϵ_r, Dissipation Factor DF and Breakdown Strength V_b of selected materials.

Material	ϵ_r 60 Hz	ϵ_r 10^6 Hz	DF 60 Hz	DF 10^6 Hz	V_b V/mil
Air	1.000585	1.000585	-	-	75
Aluminum oxide	-	8.80	-	0.00033	300
Barium titanate	1250	1143	0.056	0.0105	50
Carbon tetrachloride	2.17	2.17	0.007	<0.00004	-
Castor oil	3.7	3.7	-	-	300
Glass, soda-borosilicate	-	4.84	-	0.0036	-
Heavy Soderon	3.39	3.39	0.0168	0.0283	-
Lucite	3.3	3.3	-	-	500
Mica, glass bonded	-	7.39	-	0.0013	1600
Mica, glass, titanium dioxide	-	9.0	-	0.0026	-
Mica, ruby	5.4	5.4	0.005	0.0003	-
Mylar	2.5	2.5	-	-	5000
Nylon	3.88	3.33	0.014	0.026	-
Paraffin	2.25	2.25	-	-	250
Plexiglas	3.4	2.76	0.06	0.014	-
Polycarbonate	2.7	2.7	-	-	7000
Polyethylene	2.26	2.26	<0.0002	<0.0002	4500
Polypropylene	2.25	2.25	<0.0005	<0.0005	9600
Polystyrene	2.56	2.56	<0.00005	0.00007	500
Polysulfone	3.1	3.1	-	-	8000
Polytetrafluoroethylene(teflon)	2.1	2.1	<0.0005	<0.0002	1500
Polyvinyl chloride (PVC)	3.2	2.88	0.0115	0.016	
Quartz	3.78	3.78	0.0009	0.0001	500
Tantalum oxide	2.0	-	-	-	100
Transformer oil	2.2	-	-	-	250
Vaseline	2.16	2.16	0.0004	<0.0001	-

The power dissipated in the capacitor is

$$P = V^2 \omega C'' = V^2 \omega C(\text{DF}) \tag{11}$$

Example

Find the real and reactive power into a ruby mica capacitor with area $A = 0.03$ m^2, and a dielectric thickness $d = 0.001$ m, if the voltage is 2000 V (rms) at a frequency $f = 1$ MHz.

$$S = V^2 \frac{\omega A}{d} \epsilon_r \epsilon_o (j + \text{DF})$$

$$= (2000)^2 \frac{2\pi(10^6)(0.03)}{0.001}(5.4)(8.854 \times 10^{-12})(j + 0.0003)$$

$$= j36040 + 10.8$$

The capacitor is absorbing 36040 capacitive VARs (Volt Amperes Reactive) and 10.8 Watts. The real power of 10.8 W appears as heat and must be removed by appropriate heat sinks.

The real power dissipation in a capacitor varies directly with frequency if the dissipation factor remains constant, and also with the square of the voltage. At low frequencies, the voltage limit is determined by the dielectric strength. At high frequencies, however, the voltage limit may be determined by the ability of the capacitor to dissipate heat. If the ruby mica capacitor in the previous example could safely dissipate only 10 W, but was to be used at 5 MHz, the operating voltage must be reduced from 2000 V to keep the losses in an acceptable range.

The dissipation factor varies significantly with frequency for some materials in Table 1. The actual variation must be determined experimentally. If interpolation or extrapolation seems necessary to find the loss at some frequency not given in the Table, the best assumption would be that DF varies linearly with $\log f$. That is, if DF $= 0.02$ at $f = 10^2$ and 0.01 at $f = 10^6$, a reasonable assumption at $f = 10^4$ would be that DF $= 0.015$.

The basic circuit model for a capacitor is shown in Fig. 1. Any conductor, whether straight or wound in a coil, has inductance, and the capacitor inductance is represented by a series inductance L_s. The effects of conductor resistance and dielectric losses are represented by a series resistance R_s. Leakage current through the capacitor at dc flows through a parallel resistance R_p. In some manufacturer's databooks, these are called ESL, ESR, and EPR, where

L_s = ESL = equivalent series inductance

R_s = ESR = equivalent series resistance

R_p = EPR = equivalent parallel resistance

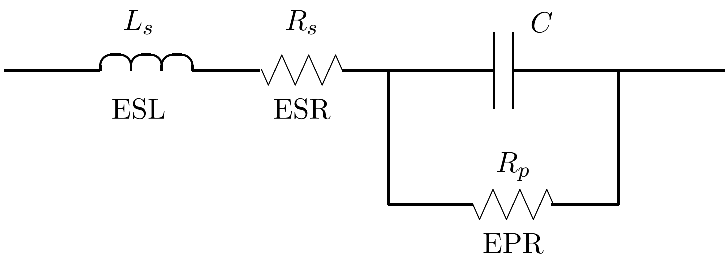

Figure 1: Equivalent Circuit for a Capacitor

This circuit indicates that every capacitor has a *self-resonant* frequency, above which it becomes an inductor. This is certain to puzzle a student making measurements on a capacitor above this frequency if the student is not aware of this fact. R_s is readily measured by applying this frequency to a capacitor, measuring the voltage and current, and calculating the ratio. The capacitive and inductive reactances cancel at the resonant frequency, leaving only R_s to limit the current. The resistance R_p will always be much larger than the capacitive reactance at the resonant frequency, so this resistance can be neglected for this computation.

The self-resonant frequency of a high capacitance unit is lower than that for a low capacitance unit. Hence, in some circuits we will see two capacitors in parallel, say a 10 μF in parallel with a 0.001 μF capacitor as shown in Fig. 2. At first glance, this seems totally unnecessary. However, the larger capacitor is used to filter low frequencies, say in the audio range, while the small capacitor filters the high frequencies which are above the self-resonant frequency of the large capacitor.

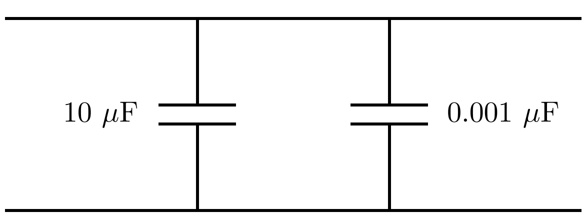

Figure 2: Capacitors in Parallel to Filter Two Different Frequencies

An example of the variation of R_s and self-resonant frequency f_{res} with capacitance value and rated voltage is given in Table 2. These are metallized polypropylene capacitors designed for switch-mode power supplies by the company Electronic Concepts, Inc. This application requires a low R_s and a high f_{res} so capacitors designed for other applications will tend to have higher values of R_s and lower values of f_{res}.

The operating frequency range will obviously be less than the self-resonant frequency. This

Table 2: Capacitor Resistance and Self-resonant Frequency for Electronic Concepts Type 5MP Metallized Polypropylene capacitors.

VDC	C	R_s	f_{res}
volts	μF	Ω	kHz
100	1	0.015	1065
100	2	0.012	703
100	5	0.010	385
100	10	0.009	248
200	1	0.020	861
200	2	0.015	609
200	5	0.011	323
200	10	0.009	200
400	1	0.019	784
400	2	0.015	511
400	5	0.010	283
400	10	0.006	200

could easily be as low as a few kHz for large motor run capacitors.

Capacitors will always be rated for a working voltage as well as a specific value of capacitance. This voltage will always be well under the breakdown voltage of the dielectric. It may be specified either as a dc voltage or an ac voltage, depending on the application. Motor run capacitors are always operated on ac, so the voltage is specified as, say, 370 or 480 VAC. They can also be operated on dc, with a dc rating of at least $\sqrt{2}$ times the ac rating. Electrolytic capacitors that can only be operated on dc will have their working voltage expressed as WVDC. This is the maximum dc voltage, plus the peak of the ac ripple voltage, that should be continuously applied to a capacitor to prevent excessive deterioration and aging. Capacitors that are sometimes used on ac, and sometimes on dc, usually have working voltages expressed as WVDC, as was the case for the polypropylene capacitors in Table 2.

We will now present some more detailed background information on several of the dielectrics shown in Table 1.

Mica is a natural material that can be easily split into thin layers. It is very stable and does not deteriorate with age. The maximum capacitance is on the order of 0.03 μF. Mica capacitors tend to be quite expensive and the larger sizes are used only in critical applications like radio transmitters.

Glass capacitors were first developed during World War II as a replacement for mica capacitors when supplies of mica were threatened. Glass capacitors exhibit excellent long-term parametric stability, low losses, and can be used at high frequencies. They are used in aerospace applications where capacitance must not vary. The maximum size is limited to about 0.01 μF.

Paper capacitors use kraft paper impregnated with non-ionized liquid electrolyte as the dielectric between aluminum foils. These capacitors can provide large capacitances and voltage ratings but tend to be physically large. The thickness of the paper layers and the particular electrolyte used can be varied to produce a wide range of electrical characteristics, which is the reason for not listing paper in the dielectrics of Table 1.

Ceramic capacitors are made with one of a large number of ceramic materials, which include aluminum oxide, barium titanate, and porcelain. These are very widely used as bypass capacitors in electronic circuits. The older style is a single layer of dielectric separating two conducting plates and packaged in a small disc. The newer style is the monolithic, which appears in a rectangular package. It consists of alternating layers of ceramic material and printed electrodes which are sintered together to form the final package. The self resonant frequency is on the order of 15 MHz for the 0.01 μF size, for a total lead length of 0.5 inch, and on the order of 165 MHz for the 0.0001 μF = 100 pF size. Ceramic capacitors are classified into Class 1 ($\epsilon_r < 600$) and Class 2 ($\epsilon_r > 600$) by the Electronics Industries Association (EIA). Class 1 ceramic capacitors tend to be larger than their Class 2 counterparts, and have better stability of values with changes in temperature, voltage, or frequency. The best Class 1 capacitor, with nearly constant characteristic, will be labeled 'C0G' using EIA designators, but is often referred to as 'NP0' which stands for 'negative-positive-zero' (temperature coefficient).

Plastic-film capacitors use extremely thin sheets of plastic film as the dielectric between capacitor plates, usually in a coil construction. They have low losses and good resistance to humidity. Four common materials are:

1. Polycarbonate
2. Polypropylene
3. Polystyrene
4. Teflon

Generally speaking, as one moves down in this list from polycarbonate to Teflon, the capacitors get larger, better, and more expensive.

Historically, low-budget Tesla coilers have used either saltwater capacitors (as Tesla himself did) or homemade rolled polyethylene and foil capacitors. The saltwater capacitors are lossy and heavy. They are one of the first components to be replaced as a new coiler grows with his hobby. The homemade foil capacitors are limited by corona onset to about 7000 V. They are oil filled, so if the container leaks or is tipped over, one has a real mess. Serious coilers would look for commercial high voltage, high current, pulse rated capacitors made by specialty companies like Maxwell. These worked fine, but were expensive if purchased new, and difficult to find on the used market.

This all changed in 1999 when the Tesla coil community moved en masse to multi mini capacitors (MMC). These are small commercial capacitors that one buys by the sackful from Digi-Key and connects in series and parallel strings to get the required ratings.

There are several different capacitor series, some of which are more suitable for Tesla coil work than others. One should look for capacitor types that are rated for "high voltage, high frequency, and high pulses". A dissipation factor of 0.1% at 1 kHz is good, as is polypropylene for the dielectric. The ECWH(V) and ECQP(U) series of capacitors are good. The Panasonic ECQ-E capacitors utilize metallized polyester (Mylar) which has a higher dissipation factor (1% at 1 kHz) than the polypropylene capacitors.

Capacitor ratings have received a great deal of attention by Tesla coilers. A given capacitor might be rated at 1600 VDC and 500 VAC. The AC rating is always for rms values, so the peak voltage would be about 700 V for a rating of 500 VAC. A capacitor in a Tesla coil primary experiences full voltage reversal, so it would seem wise to find the AC rating, multiply by $\sqrt{2}$, and divide that into the peak voltage available from the iron core transformer to find the number of series capacitors to use.

Experience of coilers, however, has been that there are sufficient safety factors built into the capacitors that using the DC rating is quite acceptable. For example, a 15 kV transformer has a peak voltage of $15\sqrt{2} = 21.2$ kV. Divide 21.2 by 1.6 for 1600 VDC capacitors to get 13.25. Use either 13 or 14 capacitors in series.

The voltage rating is not determined by the dielectric breakdown voltage as much as it is by the *Ionization or Corona Inception Level*. This refers to the level at which ionization or partial discharges can begin to occur inside a bubble of entrapped air or within an air-filled void within the solid dielectric system. If one had *perfect* dielectrics and could always exclude any entrapped air, derating for this phenomenon would not be necessary.

Corona inside a capacitor will chemically degrade the dielectric material over a period of time until the capacitor ultimately fails. Manufacturers of capacitors might select a voltage rating whereby their capacitors will last for one million hours before this occurs. In Tesla coil use, 100 hours of actual operation is a long time. This helps explain why coilers can exceed the manufacturer ratings without immediate problems.

There is evidence that operating a capacitor above its ac voltage rating will shorten its life by a factor of the overvoltage ratio raised to the 15th power. Suppose we have a capacitor rated at one million hours at 500 VAC. For economic reasons we are thinking about operating it at 700 VAC. The life reduction factor is $(700/500)^{15} = 155.57$. Dividing one million by 155.57 gives an expected life of 6428 hours or almost one year of continuous operation. This is more than adequate for most Tesla coil applications.

One brand of capacitor is the WIMA. Terry Fritz comments about them: "The switching power supplies we build have WIMAs in pulsed duty similar to that seen in Tesla coil use.

The real indicator of how long they live is how warm they get. If they heat to about 5 degrees C above the ambient, then they last forever. At about 8 degrees above ambient, we see occasional failures. At 10 degrees, they get to be a problem. In many situations, we go beyond the WIMA chart derating in pulsed applications with no problem at all, just being sure they don't get hot."

WIMA experts say that partial discharges occur only above a certain voltage level and only on ac. Frequency does not seem to be a factor. They agree that the ac rating can be exceeded in Tesla coil applications, but without a serious investigation, were mentioning factors like 1.25 to 1.5.

Some capacitors that are candidates for a MMC use actual metal foil for electrodes while others use metallized plastic. The metal foil devices are physically larger for a given rating and are much more robust in Tesla coil service.

Dielectric material, electrode thickness, and lead size all limit the rate at which charge can be added to or removed from a capacitor. This limit is expressed as the dV/dt limit. The peak current I_{peak} can be determined from the dV/dt value by the equation

$$I_{peak} = \frac{dV}{dt}C \tag{12}$$

where appropriate multipliers should be used to get everything in volts, seconds, and farads. By way of reference, a 1000 pF capacitor rated for 10,000 V/μs will withstand 10 A surges. A capacitor with a dV/dt rating less than 1000 V per 5 μs probably does not have metal foil endplates and should not be used.

The peak current that a Tesla coil primary capacitor must provide is determined by the capacitor voltage and the surge impedance. If the secondary were removed, the surge impedance would be

$$Z_s = \sqrt{\frac{L_1}{C_1}} \tag{13}$$

where C_1 is the capacitance in the Tesla coil primary and L_1 is the inductance of the primary. When the secondary is in place, the surge impedance will increase from the above value, so the peak current required from the capacitor will be less. Using the above expression for Z_s should be a worst case calculation. If it tells you that there might be 100 A flowing after the gap shorts, and your individual capacitors are rated at 10 A each, then you would need ten parallel strings. Since this figure includes some factor of safety, you might be able to get by with somewhat fewer strings. One WIMA expert said that the maximum amperage rating is not as critical as the Corona Inception Level mentioned above.

The polypropylene capacitors useful for Tesla coil work have a self-healing mode. A partial discharge will eventually cause a local failure. A burst of energy through this short will vaporize everything around the short, effectively removing the short. The capacitor continues to operate, but with a slightly lower capacitance due to some of the electrodes being removed. It is a good idea to carefully measure the capacitance of the MMC periodically. A decrease of even 1% could indicate that the capacitors are being stressed, and that total failure is possible.

Terry Fritz reports testing some WIMA MKS 4 (K4) capacitors rated at 1 μF and 400 VDC on a large DC supply. These are metallized polypropylene dielectric capacitors that are encapsulated in epoxy filled 3×1 cm rectangles. Starting at about 900 V there are a few little snaps, indicating that the dielectric had punched through and the arc blasted the thin metal layer on the other side. Going to 1200 V gave a few more snaps. Around 1500 V there were many snaps and the capacitor shorted out.

When a capacitor cleanly blows apart, that is a sign that they were destroyed by over voltage. When a capacitor puffs up and looks melted and burned, that is from too much current. It is always a good idea to think about possible failure modes, and put the capacitors in appropriate enclosures to protect bystanders.

I had a student build a small design project once, that illustrates this point. He used an electrolytic capacitor rated at about 25 VDC in some circuit for 120 VAC usage, and I did not notice it before testing. He was proudly demonstrating performance, using a variac to bring up the voltage. At about 90 V, the electrolytic capacitor exploded with a noise somewhere between a pistol and a small shotgun. The contents of the capacitor, about a cubic centimeter of plastic fibers, hit the student in the middle of the chest. He was not injured, but for a brief moment he thought he had been killed by this exploding capacitor. I suspect he still pays more attention to capacitor voltage ratings than most people.

Many coilers recommend placing resistors across the MMC to bleed off the charge after power is removed. This can eliminate some very unpleasant surprises when making adjustments between runs. One can get true high voltage resistors but they are expensive. Most use a long chain of ordinary resistors therefore. Digi-Key sells 0.5 W carbon film resistors rated at 350 volts each for about two cents each in quantity. At this price, there is little point in not using adequate safety factors.

Suppose we feel comfortable in operating a 0.5 W resistor at 0.1 W and at the rated peak voltage of 350 V each. We solve for the resistance as

$$R = \frac{V^2}{P} = \frac{(350/\sqrt{2})^2}{0.1} = 612 \ \text{k}\Omega$$

where we round off to the nearest standard value of 620 kΩ.

The number of resistors needed for a string across say a 15 kV transformer secondary would be

$$N = \frac{15000\sqrt{2}}{350} = 60 \ \text{resistors}$$

We then check the time constant to get a measure of how long it takes to discharge a

Table 3: Dielectric Constant ϵ_r' and Dissipation Factor DF of Water.

		Frequency in Hz			
Temperature		10^5	10^6	10^7	10^{10}
1.5^oC	ϵ_r'	87.0	87.0	87	38
	DF	0.1897	0.01897	0.00195	1.026
25^o	ϵ_r'	78.2	78.2	78.2	55
	DF	0.3964	0.03964	0.00460	0.545
85^oC	ϵ_r'	58	58	58	54
	DF	1.2413	0.12413	0.01259	0.259

capacitor chain. If the capacitance happened to be 27 nF, the time constant would be

$$\tau = RC = (60)(620000)(27 \times 10^{-9}) = 1 \text{ second}$$

A capacitor will be mostly discharged after five time constants or 5 seconds in this example.

There are obviously many other design possibilities. If we assumed the resistors would accept a higher voltage before arcing over, then we would want to use higher ohm values to keep heating within bounds. Terry Fritz tested some Yageo 10 MΩ 0.5 W carbon film resistors to see where they would actually fail. At 4000 V the resistor started to turn brown and smoke a little. This should be expected since it is now drawing 2 W, four times its power rating. At 5300 V, it gave a very satisfying crack and arced along its outside surface. After that, the 2 MΩ resistor measured 11.1 MΩ. So a resistor specified at 350 V actually failed at 5300 V, a safety factor of 15. Terry sees no problem with using these 0.5 W resistors up to 1000 V if the resulting wattage is not excessive.

2 Dielectric Loss in Water

Water did not appear in Table 1 because it is so much different from other substances that it needs special treatment. The relative permittivity ϵ_r' and the dissipation factor DF are shown in Table 3 for different values of temperature and frequency.

The columns for 10^5 and 10^6 Hz (100 kHz and 1 MHz) would bracket the range of operation for most Tesla coils. Only the very large coils operate below 100 kHz and the very small coils operate above 1 MHz. The column for 10^7 Hz was included to show that relative permittivity does not change between 10^6 and 10^7 Hz and that the dissipation factor continues the same decline as between 10^5 and 10^6 Hz. As frequency increases to 10^{10} Hz (the general range of microwave ovens) the relative permittivity decreases somewhat while the dissipation factor becomes quite large. It is obvious that heating food with radio waves is much more effective at 10^{10} Hz than at 10^7 Hz.

At Tesla coil frequencies, the dissipation factor $(\mathrm{DF})_2$ at some frequency f_2 can be found from the listed value $(\mathrm{DF})_1$ at frequency f_1 by

$$(\mathrm{DF})_2 = (\mathrm{DF})_1 \frac{f_1}{f_2} \tag{14}$$

3 Dielectric Losses of a Tesla Coil

There are dielectric losses primarily in two places in a Tesla coil, the coil form and the insulation around the conductor. There may also be some loss in the air surrounding the coil, in the soil, and in insulating supports near the coil, but the ones of most interest seem to be the coil form and the wire insulation. We saw in Eq. 11 that the loss in a capacitor is equal to the square of the applied voltage times the radian frequency times the capacitance times the dissipation factor. We can make reasonable estimates for voltage, frequency, and DF, but what about the capacitance?

A cross section of the coil is shown in Fig. 3.

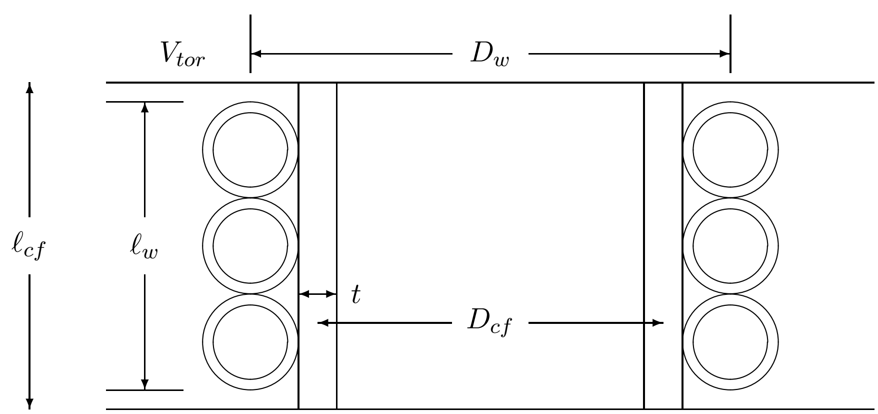

Figure 3: Cross Section of Tesla Coil

We show three turns of the several hundred that are typically used. This is a close wound coil where the conductors are immediately adjacent to one another. Space wound coils usually use a spacer of the same size as the conductor, which would result in a cross section showing half as many conductors in a given winding length. The coil diameter is D, the winding length is ℓ_w and the coil form length is ℓ_{cf}. The wall thickness of the coil form is t. At the top of the coil is a toroid with voltage V_{tor}, while at the bottom the voltage is zero. To make life a little simpler, we assume the toroid can be represented by a flat sheet on top the coil form, and that the zero potential ground plane is directly under the coil.

Most of the dielectric between toroid and ground plane is air, but a small portion is the coil form. We can think of the total capacitance between toroid and ground plane as a coil form capacitance C_{cf} in parallel with an air dielectric capacitor and a winding insulation capacitance C_{wi}. If the parallel plate capacitor formula is valid, we can quickly write an expression for the coil form capacitance.

$$C_{cf} = \frac{\epsilon \pi t D_{cf}}{\ell_{cf}} \tag{15}$$

As stated earlier, the expression is valid when fringing can be ignored. This condition is true when the separation is small compared with the area but here we have a small area and a large separation. So we go back and examine the situation for which fringing is negligible. In any capacitor, there are electric field lines directed from the positive to the negative plate, and equipotential lines (lines or surfaces of constant potential) which intersect each other at right angles. The areas enclosed by these lines are called curvilinear squares. The sides may be curved and of different lengths, but the four corners are all right angles. In a parallel plate capacitor, the equipotential lines are equally spaced straight lines, as are the electric field lines, so the curvilinear squares become true squares. It turns out that inside the coil form of Fig. 3 that electric field lines are mostly straight down and the equipotential lines are nearly equally spaced.

For the equipotential lines to be exactly equally spaced, the voltage must increase linearly from bottom to top. The actual mathematical form of the increase is probably not quite linear, as shown when the turn-to-turn voltage of the top few turns, for example, exceeds the dielectric strength of the insulation and turn-to-turn sparking occurs. But it will not be vastly different from linear, either, so the parallel plate formula should give the proper order of magnitude. I would be surprised if it were in error by more than 20% or 30%.

A circuit model for the equivalent resistances that represent losses in the coil form and in the winding insulation is shown in Fig. 4. The capacitances C_{cf} and C_{wi} are in parallel with other capacitances (not shown) such that the total capacitance is C_{tc}. The power dissipated in the coil form is given by

$$P_{cf} = V_{tor}^2 \omega C_{cf} (\mathrm{DF})_{cf} \tag{16}$$

and the equivalent resistance is given by

$$R_{cf} = \frac{V_{tor}^2}{P_{cf}} = \frac{1}{\omega C_{cf} (\mathrm{DF})_{cf}} \tag{17}$$

Example: What is the power loss and equivalent coil form resistance in a PVC coil form that is 1 m in length, 3 mm wall thickness, and diameter of 200 mm, at a frequency of 200 kHz if the top voltage is 500,000 V? Assume $\epsilon_r = 3.0$ and DF = 0.015.

$$C_{cf} = \frac{\epsilon_o \epsilon_r D_{cf} t}{\ell_{cf}} = \frac{(8.854 \times 10^{-12})(3.0)\pi(0.200)(0.003)}{1} = 5.01 \times 10^{-14} \quad \text{F}$$

$$P_{cf} = V_{tor}^2 \omega C_{cf}(\text{DF}) = (500,000)^2 2\pi(200,000)(5.01 \times 10^{-14}(0.015) = 236 \ \text{W}$$

$$R_{cf} = \frac{1}{\omega C_{cf}(\text{DF})_{cf}} = \frac{1}{2\pi(200,000)(5.01 \times 10^{-14})(0.015)} = 1.06 \times 10^9 \quad \Omega$$

To get the toroid to this voltage probably requires an input power of 10 kW or more, so 236 W is not a large loss, percentage wise. On the other hand, the coil form would soon melt if this power were applied continuously. If maximum efficiency is desired, one can always go to a polyethylene coil form, where DF is less than 0.0002

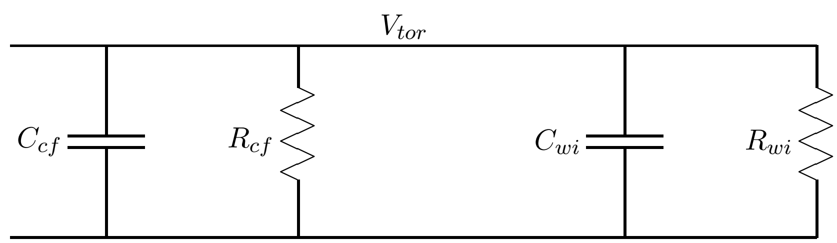

Figure 4: Equivalent Resistances for Coil Form and Wire Insulation Losses.

We turn now to the consideration of the dielectric loss in the winding insulation. Most Tesla coils are wound with magnet wire, a wire in wide use for winding motors and transformers. It has a dielectric coating that is tough both mechanically and thermally. It will withstand temperatures well above 100^{o}C. Unfortunately, it is not particularly low loss at Tesla coil frequencies. Dissipation factors are similar to PVC or nylon. The coating called Heavy Soderon actually has a thin layer of nylon on the surface, to help withstand mechanical abuse.

I have observed that the losses of tight wound magnet wire coils sometimes increase dramatically with humidity, even at input voltages far below those necessary for breakout. It seems that some magnet wire coatings can absorb moisture, and also some coil forms. I bought two plastic barrels at a recycling place that I thought were polyethylene. Whatever they were, the input resistance of the coil could change by a factor of two or more as humidity changed from 25 to 100%. On the other hand, a coil of some magnet wire on a PVC form would see little change in input resistance, probably less than 10%. So moisture in or on the wire insulation and the coil form certainly has the capability of increasing losses.

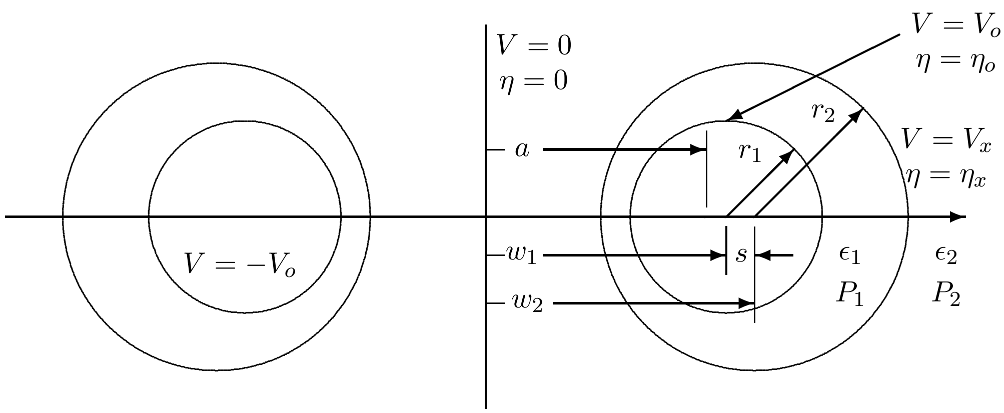

Figure 5: Two Adjacent Turns of Magnet Wire

The basic geometry that we will try to solve for power loss is shown in Fig. 5.

This figure shows two adjacent turns of a space wound coil. The conductor has radius r_1 and is surrounded by a dielectric of permittivity ϵ_1. The turns are then contained in a dielectric of permittivity ϵ_2, where usually $\epsilon_2 = \epsilon_o$. However, if the winding has been coated with some substance like polyurethane, then ϵ_2 will be different from that of air.

Like many problems in electromagnetics, there is no exact analytic formula for capacitance of the exact structure. We have a choice of an exact solution for an approximate structure, or an approximate solution for an exact structure, or combinations thereof. Fig. 5 shows the parameters for a structure that approximates our coil winding, and has an exact analytic solution. The advantage of an analytic solution is that we can easily see which factors are the most important. Questions about the effect of wire size, insulation thickness, and conductor spacing can probably be answered correctly even if the answers differ from the correct values by 20% or more.

The geometry in Fig. 5 is for a parallel conductor transmission line. The conductor on the right is at potential $V = V_o$ and the one at the left at potential $V = -V_o$. The vertical plane between the conductors is then at potential $V = 0$, by symmetry. The potential is $V = V_x$ (a constant) on the outside of the dielectric layer. If the dielectric is eccentric, then the surfaces lie on the surfaces of constant η of the bycylindrical coordinate system [2, pages 361-366]. There are three factors that make this an approximate geometry: First, the actual dielectric is centered rather than eccentric. Second, the magnet wire will lie against a coil form, which will affect the electric field lines and the capacitance. And third, there are additional conductors on either side of the two shown, which also affect field lines and capacitance. However, I think this geometry is the best that we can do, if we want an analytic solution.

The capacitance between the conductor on the right and the outer surface of its dielectric is

$$C_1 = \frac{2\pi\epsilon_1\ell}{\eta_o - \eta_x} \tag{18}$$

and the capacitance between the outer surface and the $V = 0$ plane is

$$C_2 = \frac{2\pi\epsilon_2\ell}{\eta_x} \tag{19}$$

where ℓ is the circumference of one turn of the winding. To get the power dissipation, we need the voltage $V_o - V_x$ across ϵ_1 and the voltage V_x across ϵ_2. The solution process is fairly standard for Laplace's Equation. We write general solutions for Laplace's Equation in both dielectrics.

$$V_1 = A_1 + B_1\eta \qquad \text{in} \quad \epsilon_1 \tag{20}$$

$$V_2 = A_2 + B_2\eta \qquad \text{in} \quad \epsilon_2 \tag{21}$$

The boundary equations are

$$V = V_o \qquad \text{at} \quad \eta = \eta_o \tag{22}$$

$$V = 0 \qquad \text{at} \quad \eta = 0 \tag{23}$$

$$V_1 = V_2 \qquad \text{at} \quad \eta = \eta_x \tag{24}$$

The fourth boundary equation is that the normal electric flux density is continuous on the $\eta = \eta_x$ surface.

$$\epsilon_1 \frac{\partial V_1}{\partial \eta} = \epsilon_2 \frac{\partial V_2}{\partial \eta} \tag{25}$$

After some algebra, we find

$$V_1 = \frac{V_o(\eta_x(\epsilon_1 - \epsilon_2) + \epsilon_2\eta}{\eta_o\epsilon_2 + \eta_x(\epsilon_1 - \epsilon_2)} \tag{26}$$

$$V_2 = \frac{V_o\epsilon_1\eta}{\eta_o\epsilon_2 + \eta_x(\epsilon_1 - \epsilon_2)} \tag{27}$$

We can find V_x by substituting $\eta = \eta_x$ in either of the last two equations. The power dissipations in the two regions are

$$P_1 = (V_o - V_x)^2 \omega C_1 (DF)_1 = \frac{V_o^2 \epsilon_2^2 (\eta_o - \eta_x)\omega(2\pi\epsilon_1\ell)(DF)_1}{[\eta_o\epsilon_2 + \eta_x(\epsilon_1 - \epsilon_2)]^2} \tag{28}$$

$$P_2 = V_x^2 \omega C_2 (DF)_2 = \frac{V_o^2 \epsilon_1^2 \eta_x \omega(2\pi\epsilon_2\ell)(DF)_2}{[\eta_o\epsilon_2 + \eta_x(\epsilon_1 - \epsilon_2)]^2} \tag{29}$$

The voltage V_o is half the voltage between turns. If there are N turns and the voltage at the top of the coil is V_{tor} then

$$V_o = \frac{V_{tor}}{2N} \tag{30}$$

The total power P_{wi} dissipated in the wire insulation and in the gap between the turns is

$$P_{wi} = 2N(P_1 + P_2) \tag{31}$$

The relationships between the variables in Fig. 5 are

$$w_i = \sqrt{a^2 + r_i^2} \tag{32}$$

where $i = 1$ or 2.

$$\eta_o = \sinh^{-1}\left(\frac{a}{r_1}\right) \tag{33}$$

$$\eta_x = \sinh^{-1}\left(\frac{a}{r_2}\right) \tag{34}$$

$$s = w_2 - w_1 = \sqrt{a^2 + r_2^2} - \sqrt{a^2 + r_1^2} \tag{35}$$

We start with a table lookup to find the radius r_1 of the bare magnet wire (without insulation). Then we use $w_1 = r_1+$ build + space (if any), where build refers to the thickness of the dielectric on the magnet wire, and solve for a and η_o. Then it gets a little tricky. Do we use an r_2 such that the minimum distance between the eccentric cylinders is the build, or the average distance, or something in between? My first impulse is to use the minimum distance, since most of the 'action' is between the two conductors, so we need the most accurate description of dielectric in that region. That is, I would write

$$r_2 - s = r_1 + \text{build} \tag{36}$$

This is a transendential equation in one unknown, r_2. There is no simple closed form solution, but if one plugs in values for a and r_1, it is not difficult to get a numeric value for r_2. Note that if the dielectrics are touching, then r_2, s, and w_2 all go to ∞. The model requires that $P_2 = 0$ for this case and P_1 is the dissipation in ϵ_1 if ϵ_1 fills the entire space.

Example: Find the dielectric losses in three different coils built on the same coil form used in the previous example. The winding length is one meter, the coil diameter is 200 mm, the top voltage is 500,000 V, and the toroid size is adjusted as necessary to get a frequency of 200 kHz. We want to compare the cases: (a) a 16 gauge coil tight wound, (b) a 16 gauge coil with a small air gap, 2 mils, between windings, and (c) a 22 gauge coil space wound with the same number of turns as the 16 gauge tight wound coil. These cases all have dry air as the surrounding dielectric. Then we want to examine a fourth case of the same geometry as (b) but with water condensed on the magnet wire surface. We are a long way from having a good model for this situation, but we might get some interesting results if we assume that the 2 mil gap is filled with 10% water and 90% air and use interpolated values for permittivity and dissipation factor. The magnet wire dielectric is Heavy Soderon with thickness 1.65 mils for either wire size. Results are shown in Table 4.

Table 4: Dielectric Loss Example

wire size	16	16	22	16
r_1, mils	25.4	25.4	12.65	25.4
space, mils	0	1	12.85	1
w_1, mils	27.05	28.05	27.15	28.05
N, turns	725	700	725	700
L, mH	19.0	17.7	19.0	17.7
a, mils	9.303	11.901	24.023	11.901
ϵ_{r1}	3.39	3.39	3.39	3.39
ϵ_{r2}	1	1	1	7.82
$(\text{DF})_1$	0.03	0.03	0.03	0.03
$(\text{DF})_2$	0	0	0	0.198
r_2	∞	70.32	16.03	70.32
η_o	0.3585	0.4529	1.400	0.4529
η_x	0	0.1684	1.194	0.1684
V_o	345	357	345	357
P_1	2.36	0.352	0.010	2.017
P_2	0	0	0	3.416
$2NP_1$	3423	493	14	2824
$2NP_2$	0	0	0	4782

We see a loss of 3423 W for the first case. This seems high, but the input power to the coil at the moment the top voltage is 500 kV is probably at least 10 kW and perhaps well over 100 kW, so this is not impossible. We also note that small winding irregularities lower this number significantly.

If the average gap between windings is only 2 mils, the loss drops from 3423 to 493 W. It is difficult for amateurs to wind coils with no gap at all, so the effective loss is most likely under 1 kW. If we go to a true space wound coil, as for in the 22 gauge shown in the third column, the losses drop to only 14 watts. If we planned to operate the coil in CW mode, or for very extended periods in disruptive mode, we may want to consider space winding just to keep the coil temperature rise low.

When 10% water is added to the dielectrics for the case of a 2 mil gap, the losses increase dramatically. They jump from 493 W to $2824 + 4782 = 7606$ W. These are not precise numbers, but indicate the trend that moisture increases coil losses substantially.

4 DC Capacitors

In this section we will examine the dc capacitors used to convert time varying voltages into steady dc, called filter capacitors. They usually only experience one polarity of voltage, hence can be of the *polarized* type. Aluminum oxide electrolytic and tantalum capacitors are of this type. The exact value of a capacitor used for filtering is not very critical. One may see tolerances as high as −20% to +80%. That is, a 100 μF capacitor would have an actual value between 80 and 180 μF with this tolerance. Similarly, capacitance variation with temperature is not critical.

The most important goal is a high capacitance in a small volume. We accomplish this by using a material with a high dielectric constant in a structure with a large area and thin dielectrics. One ingenious solution to this goal is the aluminum electrolytic capacitor. This consists of a ribbon of aluminum foil on which a thin film of aluminum oxide has been formed electrochemically, and a water based electrolyte fluid which acts as the opposing plate. A second ribbon of aluminum foil is used to make electrical connection to the electrolyte fluid. The two ribbons of aluminum foil are mechanically separated by a ribbon of porous paper, as shown in Fig. 6. The three ribbons are rolled up and placed in a can filled with the liquid electrolyte.

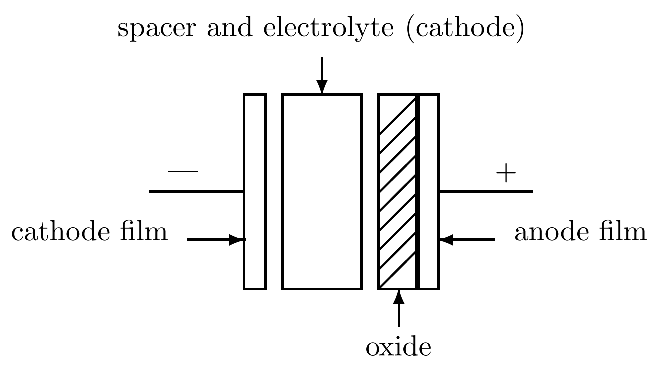

Figure 6: Cross Section of Aluminum Electrolytic Capacitor

The oxide coated foil is the positive plate, called the anode. The aluminum oxide film is the dielectric, and the fluid electrolyte is the negative plate, called the cathode. The second aluminum ribbon serves only to make good electrical connection to the cathode. It is usually bonded to the aluminum can that houses the capacitor.

The oxide dielectric has a thickness on the order of 0.01 μm. The plate separation d has therefore been reduced to a very small value. Also, aluminum oxide has a dielectric constant in the range of 8 to 11, which is relatively high for a dielectric. This combination allows a very high capacitance for a small volume.

The oxide is grown by placing the aluminum foil in an electrolyte solution and applying a dc voltage between the aluminum and the electrolyte. The resulting oxide layer has a thickness proportional to the applied voltage, about 0.0014 μm per volt at room temperature. The voltage at which the oxide is grown must be considerably higher than the proposed operating voltage to provide adequate dielectric strength. The leakage current increases rapidly as the operating voltage approaches the forming voltage value.

The film of aluminum oxide has the property of diode action. That is, current can flow one way through the anode but not the other. The aluminum electrolytic is therefore limited to dc applications. The polarity marked on the can must be carefully observed since a negative voltage more than a volt or two will cause breakdown of the film and destruction of the capacitor. This limitation is overcome in non-polarized electrolytic capacitors intended for ac applications by simply using aluminum oxide layers on *both* aluminum ribbons and facing the ribbons in opposite directions. This produces the same effect as putting two diodes in series, but pointed in opposite directions.

While the capacitance per unit volume is quite high for this capacitor type, there are several disadvantages. One is that the maximum operating voltage is limited to about 450 V. This is obviously not a problem for most computer applications, but can be a limitation in situations requiring high power. Another limitation is that the dissipation factor is higher than for other capacitor types. A third limitation is that there may be significant leakage currents. That is, both R_s and R_p need to be considered in doing loss calculations. A fourth limitation is that the shelf life is somewhat limited. A capacitor with an acceptable leakage current when first made may have an unacceptable leakage current after not being used for a year or two. There is a process called *reforming* where the capacitors are heated to 85^{o}C and a voltage is applied, which can restore acceptable performance when this problem occurs.

Tantalum capacitors usually have solid electrolytes, are smaller, and have better characteristics than aluminum capacitors. The small size of tantalum capacitors is due to the high dielectric constant of the tantalum-oxide dielectric. In addition to being small, tantalum capacitors have relatively low leakage currents, good resistance to vibrations, minimal capacitance and ESR variations with changes in temperature and no major parametric changes over the life of the device.

It is interesting to compare a high quality ac capacitor with a Tantalum. According to Table 2, a 1 μF 100 V polypropylene capacitor has a value of $R_s = 0.015\ \Omega$. A MEPCO

Series 40YW Tantalum capacitor rated at 1 μF and 50 VDC has a typical R_s of 16 Ω, and is still within specification at 80 Ω. This does not mean the Tantalum capacitor is inferior to the metallized polypropylene capacitor, but rather that the Tantalum capacitor should not be used in applications requiring large ripple currents.

Example

Calculate the power dissipation in a capacitor described by $R_s = 0.1\ \Omega$, $R_p = 10^6\Omega, L_s = 10\mu$H, and $C = 400\mu$F if the applied voltage is $v = 200 + 6\sin\omega t$, with $\omega = 2\pi(120) = 744$ rad/sec. This would describe a 200 VDC supply with a 6 V ripple voltage.

We need to apply superposition to solve for the dc power dissipation and then the ac power dissipation. At dc, C represents an infinite impedance, so we have R_s, L_s, and R_p in series. This series impedance is essentially just R_p. The average power dissipated in R_p is

$$P_{dc} = \frac{V_{dc}^2}{R_p} = \frac{(200)^2}{10^6} = 0.04\text{W}$$

At 120 Hz, the capacitive reactance is a few ohms in parallel with $R_p = 10^6\Omega$, so the capacitor model essentially becomes R_s, L_s, and C in series. The series impedance of this circuit at 120 Hz is

$$Z_{ac} = R_s + j\omega L_s - \frac{j}{\omega C} = 0.1 + j0.00744 - j3.316\Omega$$

with magnitude $|Z_{ac}| = 3.310$. Obviously, the capacitive reactance would have been an excellent approximation to the total series impedance. The rms current is then

$$I_{ac} = \frac{6}{\sqrt{2}(3.31)} = 1.28\text{A}$$

The average power due to this current flowing through R_s is

$$P_{ac} = I_{ac}^2 R_s = 0.16\text{W}$$

The total power being dissipated in the capacitor is then the sum of P_{dc} and P_{ac}.

$$P_{tot} = P_{dc} + P_{ac} = 0.04 + 0.16 = 0.2\text{W}$$

We see that the ac ripple of only 6 volts causes four times the heating of the 200 volts of dc. Also the capacitor leads must be built to carry a current of 1.28 A. This would not be a problem, but if the capacitance were increased by a factor of ten, wire size would start to become important.

5 Pulse Capacitors

Applications for pulse capacitors include particle accelerators, metal forming, laser drivers, and X-ray generators (and Tesla coils, of course). In these situations, discharges are often highly oscillatory, and voltage reversals up to 90% can be seen. Flash tubes and electrical impulse welders have relatively longer discharge times and smaller reversals. Pulse-discharge or energy-storage capacitors are usually charged over a relatively long time and discharged in a short time (microseconds or less). Voltages and currents may be very high. The inexpensive aluminum and tantalum capacitors do not meet these requirements, so special techniques are used to fabricate these capacitors. The traditional materials used include castor oil/Kraft paper, plastic film, ceramic, and mica, although the polypropylene capacitors considered earlier in the chapter would also work. This section is included to help coilers determine the suitability of surplus pulse capacitors for Tesla coil use.

An important parameter is the pulse repetition rate. If this is faster than, say, 1 pulse/minute, heating effects become important. Overheating can shorten the lifetime of a pulse capacitor considerably.

The lifetime is typically expressed in number of pulses or *shots*. Capacitors are readily available with a lifetime of greater than 100,000 shots, with voltages up to 100,000 V and peak currents up to 50,000 A, if the peak current of the first negative peak of the decaying sinusoid is no more than 30% of the first positive peak.

There are some rather interesting limits to the speed of discharge. One is that the capacitor plates act like a parallel plate transmission line. A charged transmission can only discharge in a time equal to the two-way transit time, t, along the line where

$$t = \frac{2\ell}{c}\sqrt{\epsilon_r} \tag{37}$$

where ℓ is the greatest distance from the capacitor lead to the end of the plate, and c is the speed of light in vacuum. Light propagates at a speed of 0.3 m/ns in vacuum so it is quite possible to exceed 100 ns discharge time with a rolled parallel plate construction. A capacitor built with 0.1 μF of capacitance and 10 nH of inductance will not ring down in 100 ns because of this effect, even if that is the value predicted from the (low-frequency) circuit theory model. The only way to make a fast capacitor is to manufacture it with short sections such that this transmission line transient is much shorter than the desired lumped element discharge time.

This transmission line discharge is a good way to get a single pulse of carefully controlled voltage and period. A coaxial cable is charged to a desired voltage and then discharged through the load, say with a mercury switch. This is an inexpensive method of getting a pulse of say 500 V and 40 ns.

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