SPARKS

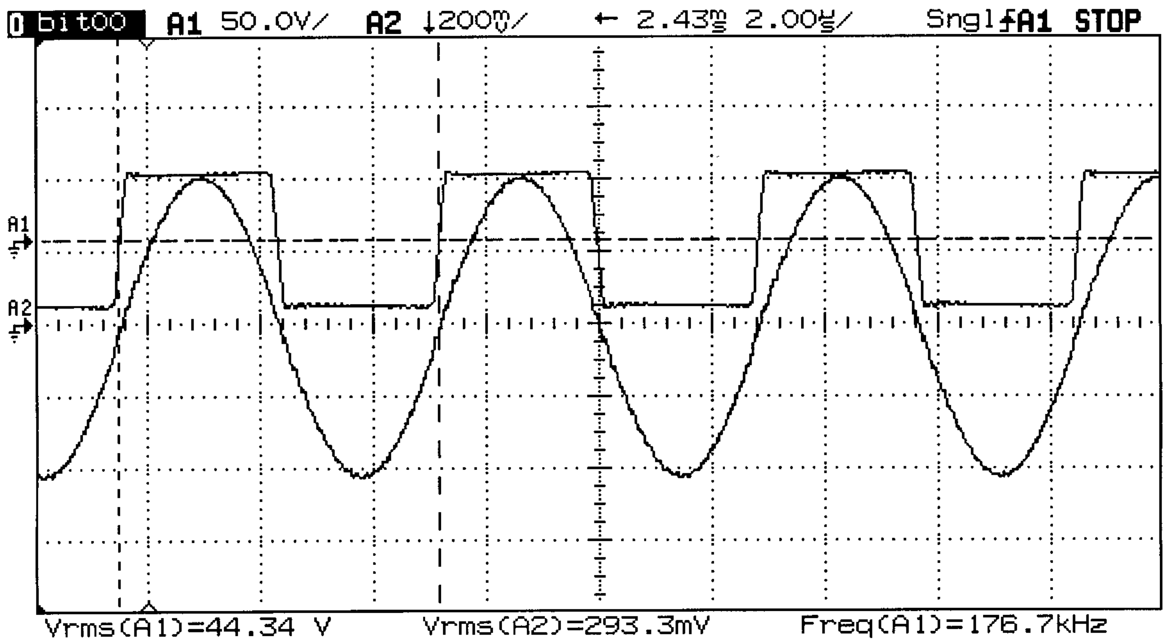

Figure 1:

Coil

The coil form for 14S was fabricated of 1/8th inch polyethylene sheet, spliced with hot glue. There were 387 turns of 14 gauge copper magnet wire space wound on the form. Coil diameter was 15.6 inches (39.64 cm) and the winding length ℓ_w was 45.9 inches (116.6 cm). Calculated inductance according to Wheeler's formula was 17.22 mH. The measured inductance was 16.94 mH with a ELC-120 meter. The Medhurst capacitance was calculated to be 23.95 pF.

The resonant frequency is then

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(17.22 \times 10^{-3})(23.95 \times 10^{-12})}} = 247.8 \text{ kHz} \quad (1)$$

Measured resonant frequency of the coil with no top load of any type was 248 kHz. All results reported here for this coil are with a half-spun aluminum toroid, nominally 24 inches outside diameter by 6 inches, for which the resonant frequency was 207 kHz.

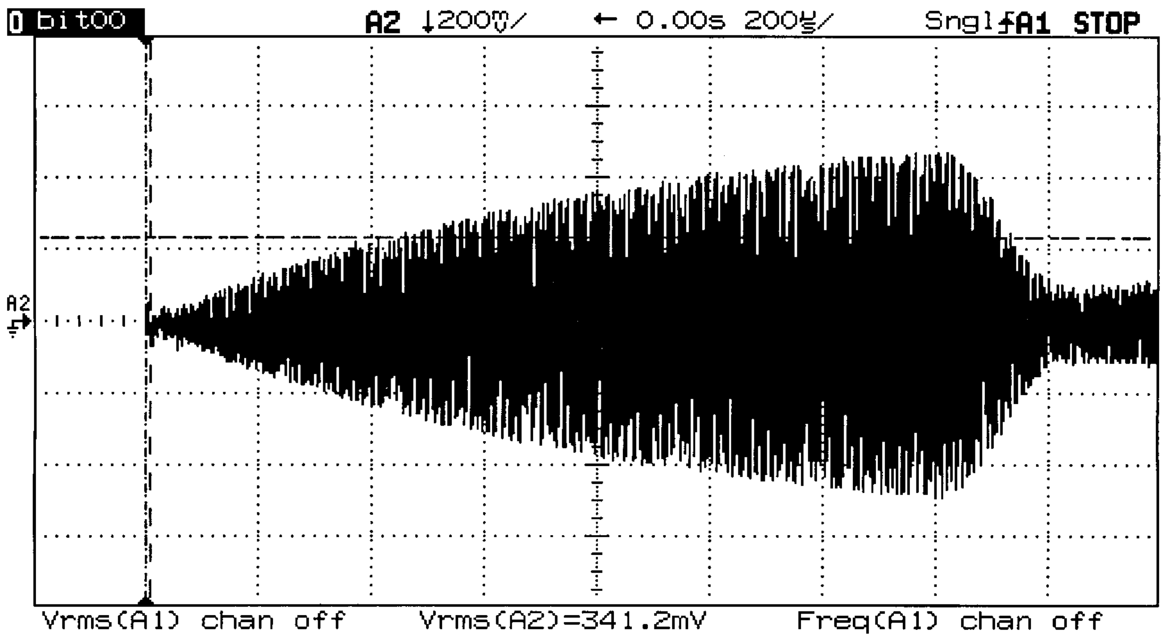

Figure 2:

We want the input impedance of the Tesla coil, the ratio of input voltage over input current. The combination of a square wave voltage and a sinusoidal current is different from what we are used to, so we need to go back to circuit theory to make sure we have all the correct multiplying factors.

A voltage square wave of value $\pm V_p$ has the same heating capability when applied to a non-inductive resistor as a dc voltage V_p, hence is said to have an rms value of $V_{ac} = V_p$. The corresponding dc current in a resistor would have an rms value of $I_{ac} = I_p$. The average power delivered to the resistor is the product of the rms voltage and the rms current,

$$P_{ave} = V_p I_p = V_{ac} I_{ac} \tag{2}$$

Suppose now we apply a sinusoidal voltage $V_p \sin \omega t$ to a non-inductive resistor. The resulting current is $I_p \sin \omega t$. The average power is

$$P_{ave} = \frac{1}{\pi} \int_0^{\pi} V_p I_p \sin^2 \theta d\theta = \frac{V_p}{\sqrt{2}} \frac{I_p}{\sqrt{2}} = \frac{V_p I_p}{2} = V_{ac} I_{ac} \tag{3}$$

When the square wave voltage produces a sinusoidal current, the integral for apparent power becomes

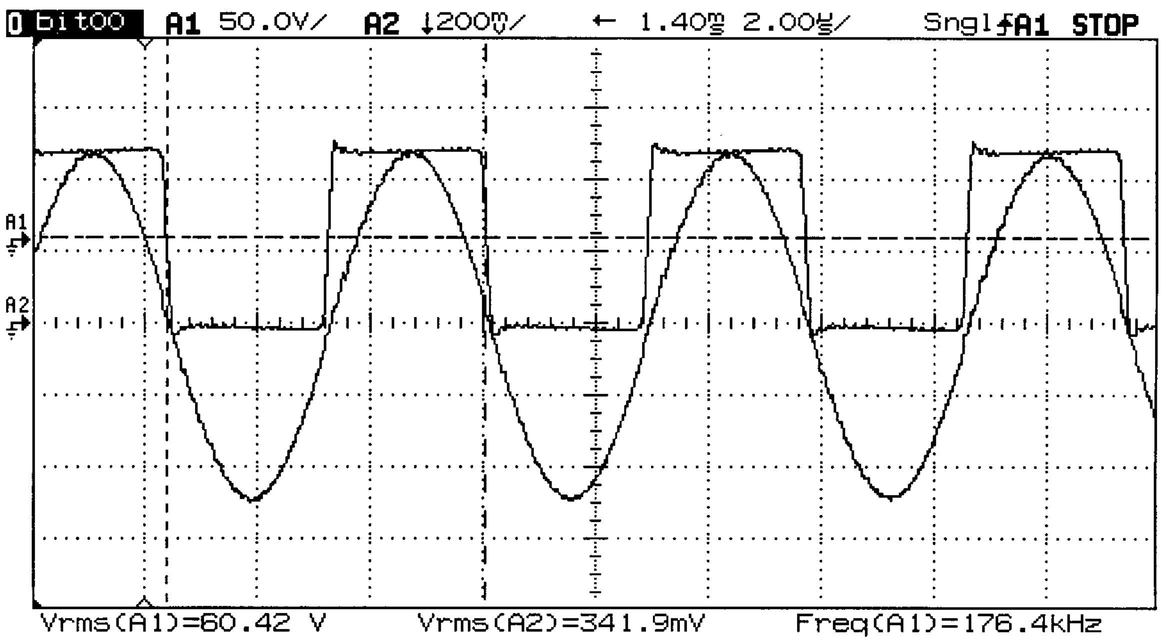

Figure 3:

$$S = \frac{1}{\pi}\int_0^\pi V_p I_p \sin\theta d\theta = \frac{2}{\pi}V_p I_p = \frac{2}{\pi}V_p(\sqrt{2}I_{ac}) = 0.9V_{ac}I_{ac} \qquad (4)$$

For this case, the apparent power is no longer the simple product of rms voltage and rms current (as for dc and single frequency sinusoids), but has a 0.9 multiplying factor. The difference is due to the fact that the square wave voltage is composed of an infinite series of harmonics (fundamental, third, fifth, etc.). Each harmonic contributes to the rms value of the square wave. The current has no harmonics, so the higher voltage harmonics do not produce any contribution to the apparent power.

If the current is in phase with the voltage then the apparent power S is equal to the average power P_{ave}. If there is a phase shift, S will be greater than P_{ave}. S is readily determined from the product of rms voltage and rms current (and appropriate multiplying constants). P_{ave} requires a wattmeter for measurement. Building an accurate wattmeter in the hundreds of kHz range is a challenge, and was not done for this research.

At resonance, when the input impedance is real, it can be defined as

$$Z_{in} = \frac{P_{ave}}{I_{ac}^2} = \frac{2\sqrt{2}V_p I_{ac}}{\pi I_{ac}^2} = \frac{2\sqrt{2}V_p}{\pi I_{ac}} = 0.9\frac{V_{ac}}{I_{ac}} \qquad (5)$$

The results are given in the following table. Units are volts, amps, watts, and volt-amps,

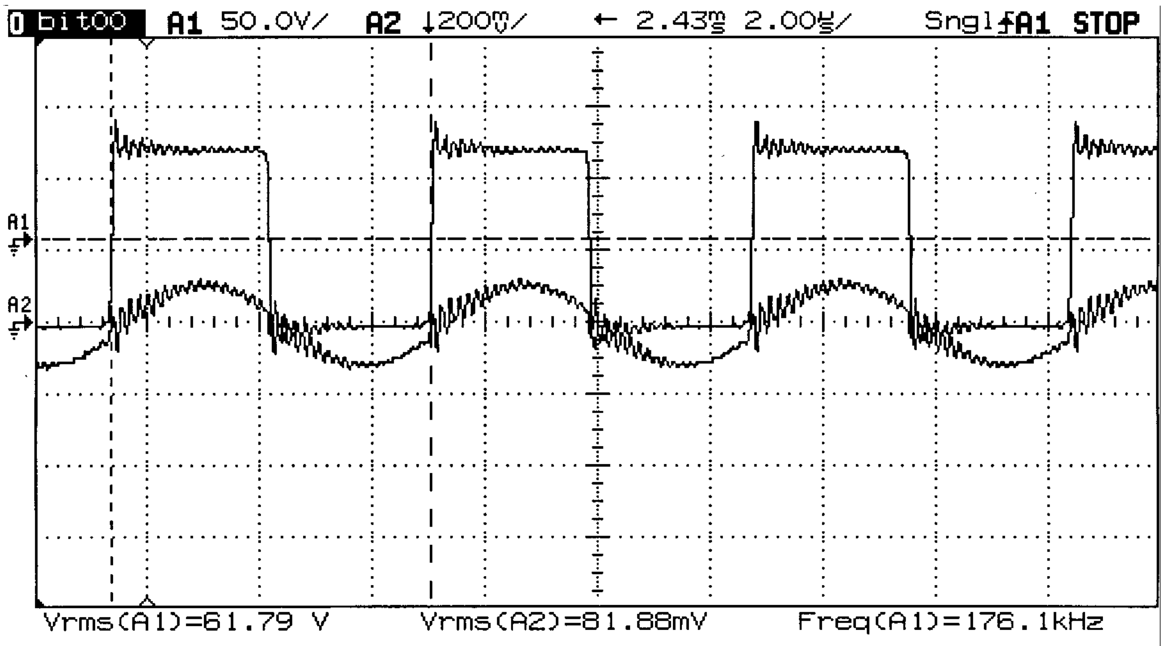

Figure 4:

except for the voltage at the Tesla coil toroid, which is in kV.

Table 1. Tesla coil measurements.

V_{dc}	I_{dc}	V_{ac}	I_{ac}	V_{top}	P_{dc}	S_{ac}	Z_{in}
40.7	0.94	37.0	1.10	21	38.3	36.6	30.3
82.5	1.94	76.4	2.25	42	160	155	30.6
121	2.85	111	3.35	58	345	335	29.9
160	3.73	145	4.43	75	599	580	29.5
218	2.28	208	2.85	47	497	533	65.7
283	2.20	270	2.75	46	623	668	88.4
346	2.15	332	2.75	46	744	822	108.6
199	2.50	190	2.78		498	475	61.6
218	2.50	214	2.78		545	534	69.4
283	2.43	274	2.75		688	677	89.8

The first four rows are for non-breakout conditions, while the last six were taken with about a 6 inch long violet plume (brush discharge) projecting from a bump on the toroid. Below breakout, the coil acts just like a monopole above a ground plane, with input impedance about 30 Ω. If the antenna were a vertical wire a quarter wavelength (90 electrical degrees) long, and had a sinusoidal current distribution, the input impedance (radiation resistance) would be 36.5 Ω. An input impedance of 30 Ω corresponds to an electrical length of about 85 degrees.

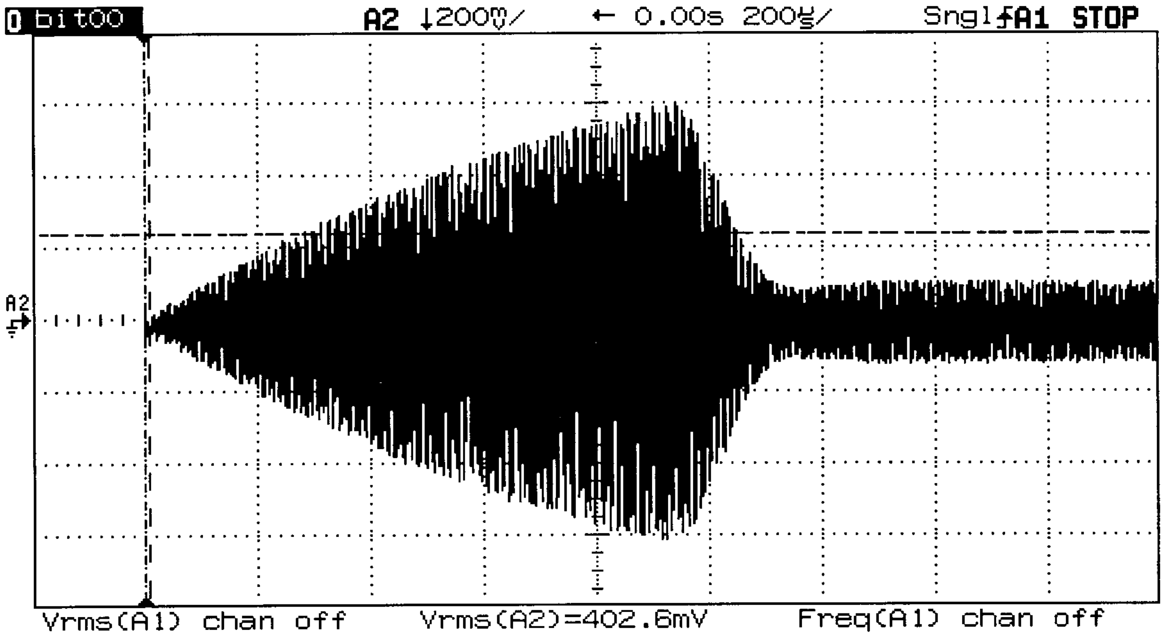

Figure 5:

The coil appears to operate as a constant-current device in breakout. Both the input dc current and the ac current into the coil stay about constant, perhaps even decreasing a little, as the ac voltage applied to the coil increases from 208 to 332 V. The top voltage appears to be proportional to the current, hence also stays about constant. The input impedance increases proportional to the voltage, in order for the current to remain constant.

It may be noticed that the apparent power S_{ac} is less than the input dc power P_{dc} below breakout (rows 1-3 in Table 1), by amounts that would be reasonable for losses in the MOSFETs, if the apparent power were numerically equal to the real power (current exactly in phase with voltage). Above breakout, apparent power is greater than the input dc power (for rows 4-7 in Table 1) and increases with applied voltage. This implies that there is now a phase shift between voltage and current. This appears to be due to the leakage reactance of the autotransformer. When the input impedance of the Tesla coil is around 30 Ω, the autotransformer is heavily loaded, and the leakage reactance is swamped. Above breakout, however, the input impedance of the Tesla coil rises rapidly, so leakage effects in the autotransformer become more noticeable. The leakage reactance is in series with a high Q RLC equivalent circuit for the Tesla coil. Increasing R changes the resonant conditions slightly, requiring some retuning for maximum current. Because of resonance, the voltage at the Tesla coil terminals will be *higher* than the voltage at the input to the leakage reactance (which cannot be measured). This resonant effect for this high Q circuit is easily enough to produce the observed increase in apparent power.

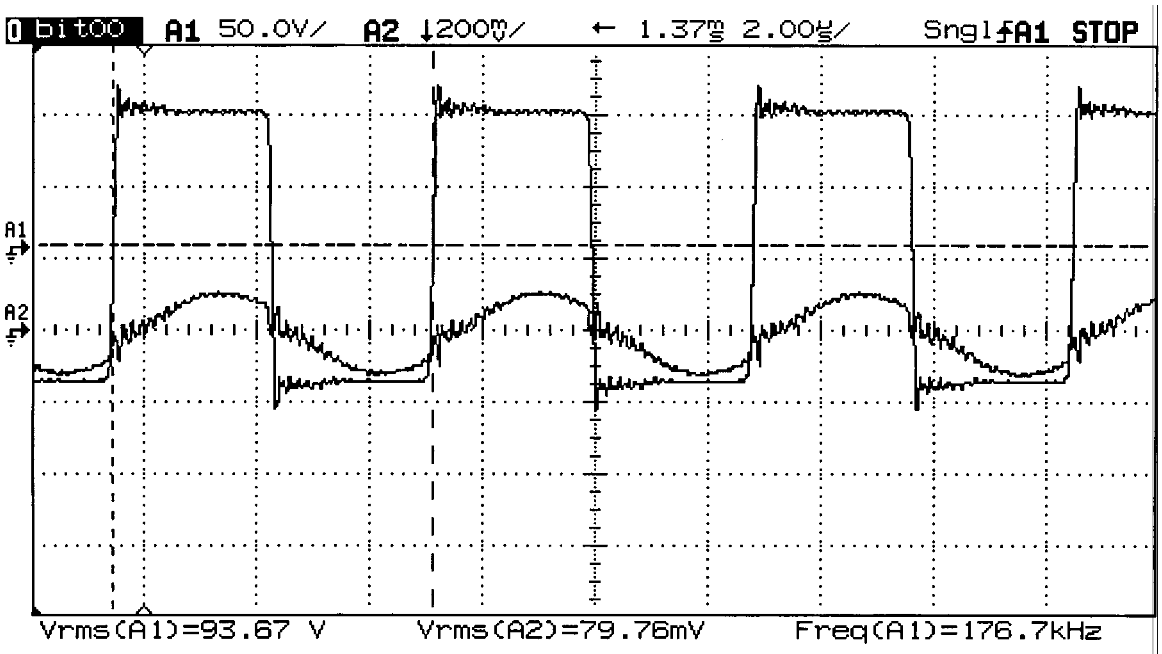

Figure 6:

To test this hypothesis, a second autotransformer was fabricated from a Phillips 4229 C81 ferrite pot core using 30 turns of 18 ga magnet wire, center-tapped. The initial permeability of this material is about 2700, hence the leakage inductance should be far less than for the first autotransformer. Data for rows 8-10 of Table 1 were taken with this autotransformer. As expected, the apparent power into the Tesla coil is now less than the average dc power into the MOSFETs. No significant changes occur in the other numbers.