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1 Calorimeter

Ohmic losses, dielectric losses, and eddy current losses in the toroid all go into heat in the Tesla coil. Eddy current losses in the building and soil, plus any radiation losses, appear as heat elsewhere. Before discovering the major effect of proximity, I was able to explain only between 10% and 30% of the input impedance with ohmic losses. Eddy current losses seemed unlikely to contribute more that a few percent to the losses. The choice seemed to reduce to dielectric losses versus radiation. In an effort to distinguish between these effects, I built a 'poor man's' calorimeter. This is basically an insulating box made of two inch thick blue styrofoam, with inside dimensions about 6 ft by 6 ft by 6.5 ft high. Seams were closed with duct tape. One panel was removable to change Tesla coil configurations inside. A small fan was mounted inside a section of 5 inch aluminum irrigation pipe and mounted vertically in a corner to provide air circulation. A Radio Shack indoor/outdoor thermometer was used to measure temperature. The outdoor temperature probe was threaded through a 10 ft length of copper water pipe and mounted inside the irrigation pipe. All pipes were electrically bonded and connected to the local ground.

Calibration was obtained by applying 60 Hz directly to the coil while measuring power with a wattmeter. Total input energy in kWh was calculated by multiplying power by time. A calibration constant could be obtained by dividing temperature rise by energy input. Each coil has a somewhat different thermal mass, so each has its own calibration constant.

One interesting observation was the difference in temperature increase between non sparking and sparking conditions. In the non sparking case, the coil gets hot and then heats the air. There is a noticeable lag between when power is applied and when temperature starts to rise. Likewise there will be a temperature overshoot for a few minutes after power is removed. In the sparking case, however, the spark heats the air directly. The coil is heated by the air, rather than the air being heated by the coil. Air temperature increases rapidly after power is applied, and decreases almost as rapidly when power is removed. The rate of decrease will change when thermal equilibrium is reached, when the coil temperature is equal to the air temperature.

For the same amount of energy dissipated inside the calorimeter, the air temperature will be higher for a longer period of time under spark conditions. This means more energy will be lost through the walls of the calorimeter. The styrofoam is rated as R10, so this wall loss becomes significant at temperature differences of more than 6 or 8^oC. One should not get too excited about small differences between spark and non spark results, therefore.

From these experimental results it was apparent that most of the losses were occurring inside the calorimeter. It was also noticed that input impedance was changing significantly from day to day. For example, on 12/29/00, I measured the input impedance of the 16 gauge coil as 87 Ω, and on 1/15/01 as 122 Ω, using identical procedures. The temperature where the coils are located was -4^oC on 12/29/00 and -1^oC on 1/15/01. This small difference would not cause such a difference in input impedance.

I then decided to check the effects of relative humidity. I placed some desiccant on the floor of the calorimeter and watched the input impedance as the space dried out. The impedance dropped from 122 Ω to 100 Ω within a few minutes, until the desiccant saturated. I then put a humidifier inside the calorimeter to raise the humidity to 100%. The impedance rose from 100 Ω to 200 Ω in 10 minutes and from 200 Ω to 300 Ω in the next 10 minutes, and finally reached 350 Ω. Moisture has a major effect on input impedance for this particular coil!

I then did the same humidifier test with the 22 gauge coil that is dimensionally the same as the 16 gauge coil. The impedance rose from 90 Ω to 150 Ω as the humidity rose to 100% for that coil.

For coil 14T, the impedance rose only from 43.7 Ω to 45.5 Ω (corrected for temperature) as the humidity rose to 100%. What can we conclude from these strange observations? As was mentioned in Chapter 6, coils 16B and 22B were wound on plastic barrels found at the recycling place. These were the only two coils that showed such a strong variation in impedance with moisture, so it seems reasonable to argue that the barrels must somehow soak up water and become much more lossy as relative humidity increases.

On the other hand, the impedance of coil 16B increases by 250 Ω while the impedance of coil 22B increases by only 60 Ω as humidity goes to 100%. The barrels appear to be identical. The two coils have identical numbers of turns and resonant frequencies. The Fraga resistance is $R_s = 51\Omega$ for 16B and 69Ω for 22B, but the measured resistance is always higher for coil 16B. Either the barrels are not actually identical or there is something about the wire on 16B that is lossier than the wire on either 14T or 22B. Both the 14 gauge and 16 gauge wires are Essex Heavy Soderon, the 14 gauge made in 1994 and the 16 gauge in 1997. Did Essex change their recipe for wire coating between 1994 and 1997? If not, then the barrels are actually different in water absorption properties or something else is happening.

I ran out of ambition before finding a firm answer to this question. I would suggest staying with PVC or polyethylene forms to help keep the water losses to a minimum. Once a coil is built and the impedance is measured, the coil can be misted with distilled water and remeasured. If the impedance changes more than perhaps 5%, expect results to vary with humidity. If this is a problem, use a different coil form and/or different wire.

2 Spark Length

Results

Figure 2 shows the voltage and current waveforms into the Tesla coil when operated at $V_{dc} =$ 100 V, which is well below breakout (no spark). The voltage wave is ideally a square wave. The ripple is due to MOSFET switching and resonances formed by stray inductances and capacitances. The main feature is that an applied square wave voltage will produce a sinusoidal current. At resonance, this current will be in phase with the voltage. The coil is high Q so tuning is very sensitive. Variation of a few hundred Hz is sufficient to get operation well away from resonance.

Figure 2. Voltage and current into Tesla coil, $V_{dc} = 100$ V.

We want the input impedance of the Tesla coil, the ratio of input voltage over input current. The combination of a square wave voltage and a sinusoidal current is different from what we are used to, so we need to go back to circuit theory to make sure we have all the correct multiplying factors.

A voltage square wave of value $\pm V_p$ has the same heating capability when applied to a non-inductive resistor as a dc voltage V_p, hence is said to have an rms value of $V_{ac} = V_p$. The corresponding dc current in a resistor would have an rms value of $I_{ac} = I_p$. The average power delivered to the resistor is the product of the rms voltage and the rms current,

$$P_{ave} = V_p I_p = V_{ac} I_{ac} \tag{1}$$

Suppose now we apply a sinusoidal voltage $V_p \sin \omega t$ to a non-inductive resistor. The resulting current is $I_p \sin \omega t$. The average power is

$$P_{ave} = \frac{1}{\pi} \int_0^{\pi} V_p I_p \sin^2 \theta d\theta = \frac{V_p}{\sqrt{2}} \frac{I_p}{\sqrt{2}} = \frac{V_p I_p}{2} = V_{ac} I_{ac} \tag{2}$$

When the square wave voltage produces a sinusoidal current, the integral for apparent power becomes

$$S = \frac{1}{\pi} \int_0^{\pi} V_p I_p \sin \theta d\theta = \frac{2}{\pi} V_p I_p = \frac{2}{\pi} V_p (\sqrt{2} I_{ac}) = 0.9 V_{ac} I_{ac} \tag{3}$$

For this case, the apparent power is no longer the simple product of rms voltage and rms current (as for dc and single frequency sinusoids), but has a 0.9 multiplying factor. The difference is due to the fact that the square wave voltage is composed of an infinite series of harmonics (fundamental, third, fifth, etc.). Each harmonic contributes to the rms value of the

Table 1. Tesla coil measurements.

V_{dc}	I_{dc}	V_{ac}	I_{ac}	V_{top}	P_{dc}	S_{ac}	Z_{in}
40.7	0.94	37.0	1.10	21	38.3	36.6	30.3
82.5	1.94	76.4	2.25	42	160	155	30.6
121	2.85	111	3.35	58	345	335	29.9
160	3.73	145	4.43	75	599	580	29.5
218	2.28	208	2.85	47	497	533	65.7
283	2.20	270	2.75	46	623	668	88.4
346	2.15	332	2.75	46	744	822	108.6
199	2.50	190	2.78		498	475	61.6
218	2.50	214	2.78		545	534	69.4
283	2.43	274	2.75		688	677	89.8

square wave. The current has no harmonics, so the higher voltage harmonics do not produce any contribution to the apparent power.

If the current is in phase with the voltage then the apparent power S is equal to the average power P_{ave}. If there is a phase shift, S will be greater than P_{ave}. S is readily determined from the product of rms voltage and rms current (and appropriate multiplying constants). P_{ave} requires a wattmeter for measurement. Building an accurate wattmeter in the hundreds of kHz range is a challenge, and was not done for this research.

At resonance, when the input impedance is real, it can be defined as

$$Z_{in} = \frac{P_{ave}}{I_{ac}^2} = \frac{2\sqrt{2}V_p I_{ac}}{\pi I_{ac}^2} = \frac{2\sqrt{2}V_p}{\pi I_{ac}} = 0.9\frac{V_{ac}}{I_{ac}} \tag{4}$$

The results are given in the following table. Units are volts, amps, watts, and volt-amps, except for the voltage at the Tesla coil toroid, which is in kV.

The first four rows are for non-breakout conditions, while the last six were taken with about a 6 inch long violet plume (brush discharge) projecting from a bump on the toroid. Below breakout, the coil acts just like a monopole above a ground plane, with input impedance about 30 Ω. If the antenna were a vertical wire a quarter wavelength (90 electrical degrees) long, and had a sinusoidal current distribution, the input impedance (radiation resistance) would be 36.5 Ω. An input impedance of 30 Ω corresponds to an electrical length of about 85 degrees.

As the voltage is increased, a point is reached where the coil goes into breakdown. For this coil and these power levels, the discharge is about six inches long into air, violet in color, and somewhat bushy (called a brush discharge). The current into the coil drops, as does the top voltage.

The coil appears to operate as a constant-current device in breakout. Both the input dc current and the ac current into the coil stay about constant, perhaps even decreasing a little, as the ac voltage applied to the coil increases from 208 to 332 V. The top voltage appears to be proportional to the current, hence also stays about constant. The input impedance increases proportional to the voltage, in order for the current to remain constant.

The resistive losses and radiated power remain about the same, in the range of 200 to 300 W, as input voltage is increased in breakout, so the additional power evidently goes into the discharge. If the total ac input power were 700 W, we would have 50 W (or less) into coil resistance, perhaps 200 W into the radiated field, and 450 W into the discharge. As the input voltage is increased even more, the resistive losses and radiated power would stay about the same while the power into the discharge increases, so the *percentage* of power supplied to the discharge increases with input voltage.

It may be noticed that the apparent power S_{ac} is less than the input dc power P_{dc} below breakout (rows 1-3 in Table 1), by amounts that would be reasonable for losses in the MOSFETs, if the apparent power were numerically equal to the real power (current exactly in phase with voltage). Above breakout, apparent power is greater than the input dc power (for rows 4-7 in Table 1) and increases with applied voltage. This implies that there is now a phase shift between voltage and current. This appears to be due to the leakage reactance of the autotransformer. When the input impedance of the Tesla coil is around 30 Ω, the autotransformer is heavily loaded, and the leakage reactance is swamped. Above breakout, however, the input impedance of the Tesla coil rises rapidly, so leakage effects in the autotransformer become more noticeable. The leakage reactance is in series with a high Q RLC equivalent circuit for the Tesla coil. Increasing R changes the resonant conditions slightly, requiring some retuning for maximum current. Because of resonance, the voltage at the Tesla coil terminals will be *higher* than the voltage at the input to the leakage reactance (which cannot be measured). This resonant effect for this high Q circuit is easily enough to produce the observed increase in apparent power.

To test this hypothesis, a second autotransformer was fabricated from a Phillips 4229 C81 ferrite pot core using 30 turns of 18 ga magnet wire, center-tapped. The initial permeability of this material is about 2700, hence the leakage inductance should be far less than for the first autotransformer. Data for rows 8-10 of Table 1 were taken with this autotransformer. As expected, the apparent power into the Tesla coil is now less than the average dc power into the MOSFETs. No significant changes occur in the other numbers.

Figure 3 shows voltage and current during discharge. V_{dc} was doubled from the data in Figure 2, from 100 to 200 V, and the corresponding input impedance was also doubled, from about 30 Ω to about 60 Ω. The Tesla coil is acting as a constant current sink.

The phase shift of the top voltage with respect to the input voltage was also checked. If the Tesla coil was acting as a quarter-wave antenna, we would expect the top voltage to lag the input voltage by 90^o, due to the time required to propagate along a transmission line. If, on the other hand, the Tesla coil is considered as the secondary of a transformer (where the primary consists of a few turns of heavy conductor), we would expect the top voltage to be in phase with the input voltage, just like any other transformer. A measured phase shift of 94^o was obtained. This does not answer all the interesting questions about electrical length. For example, if the electrical length based on input impedance is 85^o, does this imply that the phase shift from bottom to top should also be 85^o? But it supports the concept that a coil driven as a monopole above a ground plane has a phase shift from bottom to top that is close to what would be expected for a quarter-wave antenna and not that of a transformer.

A logical next step for research would be to look at the input impedance of a spiral primary driving the same coil as a secondary in a classic Tesla coil configuration. The input impedance would be on the order of one Ohm, so I_{ac} would have to be above 25 A to get enough power into the this particular coil to get breakout. The primary and secondary would need to be undercoupled so the system would have a single resonant frequency. These features raise some interesting issues, but it should be possible to make such measurements.

Figure 3. Voltage and current into Tesla coil. $V_{dc} = 200$ V.

Discussion of Results

In the first part of this paper (titled simply "Input Impedance of a Tesla Coil") some data were presented, with very little in the way of explanation. The following is a first attempt at such an explanation.

It is assumed that a direct driven (cw) Tesla coil above a ground plane can be modeled as a lumped series RLC circuit, as shown in Fig. 4. Some aspects of a Tesla coil may require transmission line theory, but many Tesla coil experts would argue that the lumped model is adequate for almost all observations, and certainly for the input impedance as measured at the base.

A classic Tesla coil, with a primary driving the secondary, has another L and another C in its model. However, when the spark quenches, the circuit of Fig. 4 applies, with V_{ac} shorted. The energy stored in L and C at the point of quenching then rings down according to the usual circuit theory equations. It appears that the R measured during cw operation should be the same as the R existing during the disruptive coil ringdown, for similar voltages and currents. If they are different, it would be nice to know why.

The coil in Part I was operated at resonance and the input voltage and current were measured. The ratio was then the input impedance at resonance, shown as R.

Below breakout, for the coil in Part 1, we measured a constant R as V_{ac} increased. The current I_{ac} therefore increased linearly with V_{ac} and the power P increased as a square law curve. At breakout, I_{ac} dropped by about a factor of two, R increased by a similar factor, and therefore $P = I_{ac}^2 R$ decreased by a factor of two. This observation can only be explained with the series model of Fig. 5.

Coil	x	ϕ	f_r	R_{dc}	R_{ac}	R_M	R_{TC}
14S	8.69	1.85	250.6	3.99	13.32	24.70	24.5
14T	8.96	3.03	266.5	4.451	15.28	46.35	43.5
16B	5.27	3.12	146.3	8.891	18.91	59.00	93.1
18B	4.18	4.25	146.8	12.90	22.50	95.61	118.0
18T	5.38	3.15	242.9	11.19	24.26	76.50	70.5
20S	4.39	1.77	256.7	18.36	33.34	59.01	43.0
20T	3.71	3.02	183.9	23.38	36.88	111.39	94.2
22T	3.81	1.62	307.7	22.40	36.10	58.38	47.0
22B	2.66	1.54	149.1	35.70	42.92	66.09	73.0

Below breakout, the current at the top of the coil is zero. This is the current actually flowing into the surrounding space. If we had an ideal quarter-wave antenna, the current distribution would be sinusoidal along the antenna, maximum at the base and zero at the top. With capacitive top loading, the current becomes more linear than sinusoidal. A linear current is plotted in Fig.8, more because it was easy to plot than because it was highly accurate.

Once the discharge starts, the current becomes greater than zero at the top. This would force some sort of redistribution of current along the coil. Since the input current was observed to decrease while the top current rises above zero, a line with less slope was drawn on Fig. 8 to represent the current for the discharge case.